kd education academy (9582701166)

Time : 6 Hour

STD 11 Maths

kd 90+ ch- 11 3-D geometry

Total Marks : 241

*** Choose the right answer from the given options. [1 Marks Each]** **[103]**

1. The ratio in which the plane 2x + 3y - 2z + 7 = 0 divides the line segment joining the points (-1, 1, 3), (2, 3, 5) is:

(A) 3 : 5 (B) 7 : 5 (C) 9 : 11 (D) 1 : 5 externally

Ans. :

d. 1 : 5 externally

2. If A = (2, -3, 1), B = (3, -4, 6) and C is a point of trisection of AB, then C_y =

(A) $\frac{11}{3}$ (B) -11 (C) $\frac{10}{3}$ (D) $\frac{-11}{3}$

Ans. :

d. $\frac{-11}{3}$

Solution:

Given, C is a point of trisection of AB.

C either divides AB in the ratio 2 : 1 or 1 : 2

Case 1: C divides in the ratio 2 : 1

The coordinates of C will be $\left(\frac{8}{3}, -\frac{11}{3}, \frac{13}{3}\right)$

Case 2: C divides in the ratio 1 : 2

The coordinates of C will be $\left(\frac{7}{3}, -\frac{10}{3}, \frac{8}{3}\right)$

either C_y = $\frac{-11}{3}$ or $-\frac{10}{3}$

3. The plane XOZ divides the join of (1, -1, 5) and (2, 3, 4) in the ratio $\lambda : 1$ then λ is:

(A) -3 (B) $\frac{-1}{3}$ (C) 3 (D) $\frac{1}{3}$

Ans. :

d. $\frac{1}{3}$

Solution:

The plane XOZ divides the join of (1, -1, 5) and (2, 3, 4) in the ratio $\lambda : 1$ i.e. y = 0 divide the join of (1, -1, 5) and (2, 3, 4) in the ratio.

$\lambda : 1 \therefore \frac{3\lambda - 1}{\lambda + 1} = 0$

$\Rightarrow \lambda = \frac{1}{3}$

4. A point C with position vector $\frac{3a+4b-5c}{3}$ (where a, b and c are non co-planar vectors) divides the line joining A and B in the ratio 2 : 1. If the position vector of A is a - 2b + 3c, then the position vector of B is:

(A) 2a + 3b - 4c (B) 2a - 3b + 4c (C) 2a + 3b + 4c (D) a + 3b - 4c

Ans. :

d. a + 3b - 4c

Solution:

a - 2b + 3c

$\frac{3a+4b-5c}{3}$

$\vec{c} = \frac{2\vec{b}+\vec{a}}{3}$

$\vec{b} = \frac{3\vec{c}-\vec{a}}{2}$

$= \frac{\left(3\vec{a}+4\vec{b}-5\vec{c}\right)-\left(\vec{a}+2\vec{b}-3\vec{c}\right)}{2}$

$= \vec{a} + 3\vec{b} - 4\vec{c}$

5. The plane. ax + by + cz + (-3) = 0 meet the co-ordinate axes in A, B, C. The centroid of the triangle is:

(A) $(3a, 3b, 3c)$ (B) $\left(\frac{3}{a}, \frac{3}{b}, \frac{3}{c}\right)$ (C) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (D) $\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$

Ans. :

d. $\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$

6. D(2, 1, 0), E(2, 0, 0), F(0, 1, 0) are mid point of the sides BC, CA, AB of ΔABC respectively, The the centroid of ΔABC is:

(A) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (B) $\left(\frac{4}{3}, \frac{2}{3}, 0\right)$ (C) $\left(-\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (D) $\left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$

Ans. :

b. $\left(\frac{4}{3}, \frac{2}{3}, 0\right)$

Solution:

Centroid of triangle coincide with the centroid of triangle formed by joing the mid-point of sides of triangle. So, centroid of $\triangle ABC$ = centroid of.

$\triangle DEF = \left(\frac{2+2+0}{3}, \frac{1+0+1}{3}, \frac{0+0+0}{3}\right) = \left(\frac{4}{3}, \frac{2}{3}, 0\right)$

7. The plane XOZ divides the join of (1, -1, 5) and (2, 3, 4) in the ratio $\lambda : 1$ then λ is:

(A) -3 (B) $\frac{1}{4}$ (C) 3 (D) $\frac{1}{3}$

Ans. :

d. $\frac{1}{3}$

Solution:

Given points are (1, -1, 5) and (2, 3, 4)Using section formula the desired points is

$= \left(\frac{2\lambda+1}{\lambda+1}, \frac{3\lambda-1}{\lambda+1}, \frac{4\lambda+5}{\lambda+1}\right)$

Since, this point lies in XOZ plane then its yy-co-ordinate should be zero.

$\Rightarrow \frac{3\lambda-1}{\lambda+1} = 0$

$\Rightarrow \lambda = \frac{1}{3}$

8. The image of the point P(1, 3, 4) in the plane 2x - y + z = 0 is:

   (A) (-3, 5, 2) (B) (3, 5, 2) (C) (3, -5, 2) (D) (3, 5, -2)

Ans. :

a. (-3, 5, 2)

9. Which octant do the point (-5, 4, 3) lie:

   (A) Octant I (B) Octant II (C) Octant III (D) Octant IV

Ans. :

b. Octant II

Solution:

Given, (-5, 4, 3) is the point

Here, the x-coordinate is negative but y and z coordinates are positive

$\therefore$ (-5, 4, 3) lie in octant II.

10. The points (5, 2, 4), (6, -1, 2) and (8, -7, k) are collinear, if k is equal to:

   (A) -2 (B) 2 (C) 3 (D) -1

Ans. :

a. -2

11. XOZ-plane divides the join of (2, 3, 1) and (6, 7, 1) in the ratio

   (A) 3 : 7 (B) 2 : 7 (C) -3 : 7 (D) -2 : 7

Ans. :

c. -3 : 7

Solution:

Let A ≡ (2, 3, 1) and B ≡ (6, 7, 1)

Let the line joining A and B be divided by the xz-plane at point P in the ratio $\lambda : 1$.

Then, we have,

$\mathrm{P} \equiv \left(\frac{6\lambda+2}{\lambda+1}, \frac{7\lambda+3}{\lambda+1}, \frac{\lambda+1}{\lambda+1}\right)$

Since P lies on the xz-plane, the y-coordinate of P will be zero.

$\therefore \frac{7\lambda+3}{\lambda+1} = 0$

$\Rightarrow 7\lambda + 3 = 0$

$\Rightarrow \lambda = \frac{-3}{7}$

Hence, the xz-plane divides AB in the ratio -3 : 7

12. Choose the correct answer.

    If a parallelopiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then the length of diagonal of the parallelopiped is:

   (A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

Ans. :

a. $2\sqrt{3}$

Solution:

Given parallelepiped passes through A(5, 8, 10) and B(3, 6, 8)

$\therefore$ Length of the diagonal,

$\mathrm{AB} = \sqrt{(5-3)^2 + (8-6)^2 + (10-8)^2} = \sqrt{4+4+4} = 2\sqrt{3}$

13. If the zx-plane divides the line segment joining (1, -1, 5) and (2, 3, 4) in the ratio p : 1 then p + 1 =

(A) $\frac{1}{3}$ (B) $1:3$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$

Ans. :

d. $\frac{4}{3}$

Solution:

Given, points are (1, -1, 5) and (2, 3, 4) since ZX-plane divides the line segment in the ratio p : 1, y-coordinate will be 0 the y-coordinate of the point dividing the line segment will be.

$= \frac{3\mathrm{p}-1}{\mathrm{p}+1} = 0,\ \mathrm{p} = \frac{1}{3}\ \mathrm{p}+1 = \frac{1}{3}+1 = \frac{4}{3}$

14. L is the foot of the perpendicular drawn from a point P(3, 4, 5) on the xy-plane. The coordinates of point L are:

(A) (3, 0, 0) (B) (0, 4, 5) (C) (3, 0, 5) (D) None of these

Ans. :

d. None of these

15. Choose the correct answer.

L is the foot of the perpendicular drawn from a point P(3, 4, 5) on the xy-plane. The coordinates of point L are:

(A) (3, 0, 0). (B) (0, 4, 5). (C) (3, 0, 5). (D) None of these.

Ans. :

d. None of these.

Solution:

We know that on the xy-plane, z = 0.

Hence, the coordinates of the points L are (3, 4, 0).

16. Find the ratio in which 2x + 3y + 5z = 1 divides the line joining the points (1, 0, -3) and (1, -5, 7):

(A) 1 : 2 (B) 2 : 1 (C) 3 : 2 (D) 2 : 3

Ans. :

d. 2 : 3

17. Three vertices of a parallelogram ABCD are A(1, 2, 3), B(-1, -2, -1) and C(2, 3, 2). Find the fourth vertex D:

(A) (-4, -7, -6) (B) (4, 7, 6) (C) (4, 7, -6) (D) None of these

Ans. :

b. (4, 7, 6)

18. If G is centroid of $\triangle \mathrm{ABC}$ then:

(A) $\vec{G}=\vec{a}+\vec{b}+\vec{c}$ (B) $\vec{G}=\frac{\vec{a}+\vec{b}+\vec{c}}{2}$ (C) $3\vec{G}=\vec{a}+\vec{b}+\vec{c}$ (D) $3\vec{G}=\frac{\vec{a}+\vec{b}+\vec{c}}{2}$

Ans. :

c. $3\overrightarrow{\mathrm{G}}=\overrightarrow{\mathrm{a}}+\overrightarrow{\mathrm{b}}+\overrightarrow{\mathrm{c}}$

Solution:

We have,

In a $\triangle \mathrm{ABC}$

$\overrightarrow{\mathrm{A}}=\overrightarrow{\mathrm{a}}$

$\overrightarrow{\mathrm{B}}=\overrightarrow{\mathrm{b}}$

$\overrightarrow{\mathrm{C}}=\overrightarrow{\mathrm{c}}$

then,

we know that,

$\overrightarrow{\mathrm{G}}=\frac{\overrightarrow{\mathrm{a}}+\overrightarrow{\mathrm{b}}+\overrightarrow{\mathrm{c}}}{3}$

$3\overrightarrow{\mathrm{G}}=\overrightarrow{\mathrm{a}}+\overrightarrow{\mathrm{b}}+\overrightarrow{\mathrm{c}}$

19. What is the distance between the points (2, -1, 3) and (-2, 1, 3):

(A) $2\sqrt{5}$ units (B) 25 units (C) $4\sqrt{5}$ units (D) $\sqrt{5}$ units

Ans. :

a. $2\sqrt{5}$ units

20. The distance of the point P(a, b, c) from the x-axis is:

(A) $\sqrt{(\mathrm{a}^2+\mathrm{c}^2)}$ (B) $\sqrt{(\mathrm{a}^2+\mathrm{b}^2)}$ (C) $\sqrt{(\mathrm{b}^2+\mathrm{c}^2)}$ (D) None of these

Ans. :

c. $\sqrt{(\mathrm{b}^2+\mathrm{c}^2)}$

Solution:

The coordinate of the foot of the perpendicular from P on x-axis are (a, 0, 0).

So, the required distance $=\sqrt{(\mathrm{a}-\mathrm{a})^2+(\mathrm{b}-0)^2+(\mathrm{c}-0)^2}$

$=\sqrt{(\mathrm{b}^2+\mathrm{c}^2)}$

21. If α,β,y are the angles made by a half ray of a line respectively with positive directions of X-axis, Y-axis and, Z-axis, then $\sin^2\alpha+\sin^2\beta+\sin^2\mathrm{y}=$

(A) 1 (B) 0 (C) -1 (D) None of these

Ans. :

d. None of these

22. Find the image of (-2, 3, 4) in the y z plane:

(A) (-2, 3, 4) (B) (2, 3, 4) (C) (-2, -3, 4) (D) (-2, -3, -4)

Ans. :

b. (2, 3, 4)

23. What is the length of foot of perpendicular drawn from the point P(3, 4, 5) on y-axis:

(A) $\sqrt{41}$ (B) $\sqrt{34}$ (C) 5 (D) None of these

Ans. :

b. $\sqrt{34}$

24. The coordinates of a point which divides the line joining the points P(2, 3, 1) and Q(5, 0, 4) in the ratio 1 : 2 are:

(A) $\left(\frac{7}{3}, 1, \frac{5}{3}\right)$ (B) (4, 1, 3) (C) (3, 2, 2) (D) (1, -1, 1)

Ans. :

c. (3, 2, 2)

Solution:

Using section formula, Coordinate of the point which divides P (2, 3, 1) and Q (5, 0, 4) in ratio.

1 : 2 is $\left(\frac{2.2+5}{2+1}, \frac{2.3+0}{2+1}, \frac{2.1+4}{2+1}\right) = \left(\frac{9}{6}, \frac{6}{3}, \frac{6}{3}\right) = (3, 2, 2)$

25. Point A is a + 2b, and a divides AB in the ratio 2 : 3. The position vector of B is:

(A) 2a - b (B) b - 2a (C) a - 3b (D) b

Ans. :

c. a - 3b

Solution:

Let us consider x be the position vector of B, then a divides AB in the ratio 2 : 3

$\mathrm{a} = \frac{2\mathrm{x}+3(\mathrm{a}+2\mathrm{b})}{2+3}$

$\Rightarrow \mathrm{x} = \mathrm{a} - 3\mathrm{b}$

26. G(1, 1, -2) is the centroid of the triangle ABC and D is the mid point of BC. If A = (-1, 1, -4) D=

(A) $\left(\frac{1}{2}, 1, \frac{-5}{2}\right)$ (B) (5, 1, 2) (C) (-5, -1, -2) (D) (2, 1, -1)

Ans. :

d. (2, 1, -1)

Solution:

Let the coordinates of D be (p, q, r)

Since, the centroid divides the line joining AD in the ratio 2 : 1 the coordinates of centroid should be,

$\left(\frac{2p-1}{3}, \frac{2q+1}{3}, \frac{2r-4}{3}\right)$

Comparing it with the coordinates of the centroid given, D (2, 1, -1).

27. If the plane 7x + 11y + 13z = 3003 meets the axes in A, B, C then the centroid of ΔABC is:

(A) (143, 91, 77) (B) (143, 77, 91) (C) (91, 143, 77) (D) (143, 66, 91)

Ans. :

a. (143, 91, 77)

28. Graph $x^2 + y^2 = 4$ in 3D looks like:

(A) Circle (B) Cylinder (C) Hemisphere (D) Sphere

Ans. :

b. Cylinder

Solution:

The given curve is $x^2 + y^2 = 4$ So x coordinate and y-coordinate are connected by $x^2 + y^2 = 4$ which is locus of a circle with radius 2 But z-coordinate can be anything,

so in three dimension the circle $x^2 + y^2 = 4$ will be stretched which will be a cylinder with radius same as the radius of the circle.

29. Find the distance between (12, 3, 4) and (4, 5, 2):

(A) $\sqrt{72}$ (B) $\sqrt{62}$ (C) $\sqrt{64}$ (D) None of these

Ans. :

a. $\sqrt{72}$

Solution:

Consider the problem,

Let the given points

A(12, 3, 4) and B(4, 5, 2)

So, distance between A and B by distance formula.

$$AB = \sqrt{(4-12)^2 + (5-3)^2 + (2-4)^2} = \sqrt{(-8)^2 + 2^2 + (-2)^2}$$

$$= \sqrt{64+4+4} = \sqrt{72}$$

So, distance between the points (12,3,4) and (4,5,2) is $\sqrt{72}$ Sq. units.

30. The ratio in which yz-plane divides the line segment joining (-3, 4, 2), (2, 1, 3) is:

(A) -4 : 1 (B) 3 : 2 (C) -2 : 3 (D) 1: 4

Ans. :

b. 3 : 2

Solution:

Let the plane divide the line in the ratio p : 1

A point that divides the line joining these 2 points in the ratio p : 1

given by $\left(\frac{2p-3}{p+1}, \frac{p+4}{p+1}, \frac{3p+2}{p+1}\right)$

Since, this point has to lie on the zy-plane. so, 2p - 3 = 0

$\Rightarrow p = \frac{3}{2}$

31. The perpendicular distance of the point P(3, 3, 4) from the x-axis is

(A) $3\sqrt{2}$ (B) 5 (C) 3 (D) 4

Ans. :

b. 5

Solution:

The perpendicular distance of the point P(3, 3, 4) from the x-axis is given by

$\sqrt{3^2+4^2}$

$= \sqrt{25}$

$= 5$

Hence, the correct answer is option (b)

32. The vector equation of a sphere having centre at origin and radius 5 is:

(A) $|r| = 5$ (B) $|r| = 25$ (C) $|r| = \sqrt{5}$ (D) None of these

Ans. :

a. $|r| = 5$

33. If A = (1, 2, 3), B = (2, 3, 4) and AB is produced upto C such that 2AB = BC then C =

(A) (5, 4, 6) (B) (6, 2, 4) (C) (4, 5, 6) (D) (6, 4, 5)

Ans. :

c. (4, 5, 6)

Solution:

Let the point C be (i, j, k)

Since, B divides AC in the ratio 1 : 2

Coordinates of B should be $\left(\frac{2+i}{3}, \frac{4+j}{3}, \frac{k+6}{3}\right)$

Comparing the values given already for B, we get, i = 4, j = 5 and k = 6

34. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$ then the value of a is:

(A) 5 (B) ± 5 (C) -5 (D) None of these

Ans. :

b. ± 5

35. The ratio in which the line joining the points (1, 2, 3) and (-3, 4, -5) is divided by the xy-plane is:

(A) 2 : 5 (B) 3 : 5 (C) 5 : 2 (D) 5 : 3

Ans. :

b. 3 : 5

36. A = (1, 1, 4) and B = (5, -3, 4) are two points. If the points P, Q are on the line AB such that AP = PQ = QB then PQ =

(A) $2\sqrt{2}$ (B) 4 (C) $\sqrt{\frac{32}{9}}$ (D) $\sqrt{2}$

Ans. :

c. $\sqrt{\frac{32}{9}}$

Solution:

$\mathrm{AB} = \sqrt{(1-5)^2+(1+3)^2+(4+4)^2}$

$\mathrm{AB} = \sqrt{(-4)^2+4^2}$

$\mathrm{AB} = \sqrt{32}$

$\mathrm{AB} = 3 \times \mathrm{PQ}, = \frac{\sqrt{132}}{3} = \sqrt{\frac{32}{9}}$

37. The points (-5, 12), (-2, -3), (9, -10), (6, 5) taken in order, form:

(A) Parallelogram (B) Rectangle (C) Rhombus (D) Square

Ans. :

a. Parallelogram

38. In a three dimensional space the equation $x^2 - 5x + 6 = 0$ represents

(A) Points. (B) Planes.

(C) Curves. (D) Pair of straight lines.

Ans. :

c. Curves.

Solution:

Since, there is only one variable in the given equation.

Also, it is quadratic equation.

Hence, It represents curves in yz plane.

39. The cartesian equation of the line is 3x + 1 = 6y - 2 = 1 - z then its direction ratio are:

(A) $\frac{1}{3}, \frac{1}{6}, 1$ (B) $\frac{-1}{3}, \frac{1}{6}, 1$ (C) $\frac{1}{3}, \frac{-1}{6}, 1$ (D) $\frac{1}{3}, \frac{1}{6}, -1$

Ans. :

a. $\frac{1}{3}, \frac{1}{6}, 1$

40. A plane intersects the co ordinate axes at A, B, C. If O = (0, 0, 0) and (1, 1, 1) is the centroid of the tetrahedron OABC, then the sum of the reciprocals of the intercepts of the plane:

(A) 12

(B) $\frac{4}{3}$ (C) 1 (D) $\frac{3}{4}$

Ans. :

d. $\frac{3}{4}$

Solution:

Let the point of intersections be,

A (a, 0, 0), B (0, b, 0) and C (0, 0, c)

The coordinates of the centroid are $\left(\frac{a}{4}, \frac{b}{4}, \frac{c}{4}\right)$

Comparing it with the coordinates given, we get

a = 4, b = 4, c = 4

$\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right) = \frac{3}{4}$

41. Find the distance between the points whose position vectors are given as follows: $4\hat{i} + 3\hat{j} - 6\hat{k}, -2\hat{i} + \hat{j} - \hat{k}$

(A) $\sqrt{65}$ (B) $\sqrt{69}$ (C) 1 (D) None of these

Ans. :

a. $\sqrt{65}$

42. If the line joining A(1, 3, 4) and B is divided by the point (-2, 3, 5) in the ratio 1 : 3, then B is:

(A) (-11, 3, 8) (B) (-11, 3, -8) (C) (-8, 12, 20) (D) (13, 6, -13)

Ans. :

a. (-11, 3, 8)

43. A = (1, -1, 2) and B = (2, 3, 7) are two points. If P, O divide AB in the ratios 2 : 3, -2 : 3 respectively then $P_x + Q_y$ =

(A) $\frac{-38}{5}$ (B) $\frac{38}{5}$ (C) $\frac{-2}{5}$ (D) $\frac{-47}{6}$

Ans. :

a. $\frac{-38}{5}$

Solution:

P divides line joining A (1, -1, 2) and B (2, 3, 7) in the ratio 2 : 3

$\therefore P_x = \frac{2\times2+3\times1}{2+3} = \frac{7}{5}$ Similarly, Q divides line joining A (1, -1, 2) and B (2, 3, 7) in the ratio -2 : 3

$\therefore Q_y = \frac{-2\times3+3\times-1}{-2+3} = -9$

$\Rightarrow P_x + Q_y = -9 + \frac{7}{5} = \frac{-38}{5}$

44. If the extremities of the diagonal of a square are (1, -2, 3 and (2, -3, 5), then the length of the side is

(A) $\sqrt{6}$ (B) $\sqrt{3}$ (C) $\sqrt{5}$ (D) $\sqrt{7}$

Ans. :

b. $\sqrt{3}$

Solution:

Length of the diagonal $= \sqrt{(2-1)^2+(-3+2)^2+(5-3)^2} = \sqrt{1+1+4} = \sqrt{6}$

$\therefore$ Length of the side $= \frac{\text{Length of diagonal}}{\sqrt{2}} = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$

45. In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are:

(A) Three mutually parallel lines

(B) Three mutually perpendicular lines

(C) Two mutually perpendicular lines and any two parallel

(D) None of these

Ans. :

b. Three mutually perpendicular lines

Solution:

In three dimensions, the coordinate axes, i.e. x, y and z axes of a rectangular cartesian coordinate system are three mutually perpendicular lines. The word rectangular is used to indicate perpendicularity among the axes.

46. An equation of sphere with centre at origin and radius r can be represented as:

(A) $x^2 + y^2 + z^2 = r$

(B) $x^2 + y^2 + z^2 = r^2$

(C) $x^2 + y^2 + z^2 = 2r^2$

(D) None of the above

Ans. :

b. $x^2 + y^2 + z^2 = r^2$

Solution:

Sphere is locus of a point in 3D whose distance from a fixed point (center) is constant (radius)

$\Rightarrow \sqrt{(\mathrm{x}-0)^2+(\mathrm{y}-0)^2+(\mathrm{z}-0)^2}$

$=|\mathrm{r}| \Rightarrow \mathrm{x}^2+\mathrm{y}^2+\mathrm{z}^2=\mathrm{r}^2$ square both sides.

47. The position vectors of the four angular point of a tetrahedron OABC are (0, 0, 0), (0, 0, 2), (0, 4, 0) and (6, 0, 0) respectively. Find the coordinates of cenroid:

(A) $\left(2, \frac{4}{3}, \frac{2}{3}\right)$

(B) $\left(\frac{6}{4}, 1, \frac{2}{4}\right)$

(C) (0, 0, 0)

(D) None of these

Ans. :

b. $\left(\frac{6}{4}, 1, \frac{2}{4}\right)$

Solution:

Angular points of tetrahedron OABC are.

(0, 0, 0), (0, 0, 2), (0, 4, 0), (6, 0, 0) To find the coordinates of the centroid of the tetrahedron whose vertices are

(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) the centroid is

$\left(\frac{x_1+x_2+x_3+x_4}{4}\right), \left(\frac{y_1+y_2+y_3+y_4}{4}\right), \left(\frac{z_1+z_2+z_3+z_4}{4}\right)$

Now, substituting the values we get

$\left(\frac{0+0+0+6}{4}\right), \left(\frac{0+0+4+0}{4}\right), \left(\frac{0+2+0+0}{4}\right)$

$\therefore$ The coordinates of the centroid are $\left(\frac{6}{4}, 1, \frac{2}{4}\right)$

48. The perpendicular distance of the point P(6, 7, 8) from xy-plane is

(A) 8 (B) 7 (C) 6 (D) 10

Ans. :

a. 8

Solution:

The distance of the point P(6, 7, 8) from the xy-plane is equal to the z-coordinate of the point.

Here, the value of z-coordinate is 8.

Hence, the correct answer is option (a).

49. Area of quadrilateral whose vertices are (2, 3), (3, 4), (4, 5) and (5, 6), is equal to:

(A) 0 (B) 4 (C) 6 (D) None of these

Ans. :

a. 0

50. The point A(1, -1, 3), B(2, -4, 5) and C(5, -13, 11) are:

(A) Collinear (B) Non-collinear

(C) Do not say anything (D) None of these

Ans. :

a. Collinear

51. Let $P(x, y, z)$ be a point in the first octant, whose projection in the xy-plane is the point Q. Let $OP = \gamma$; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is :

(A) $\gamma\sqrt{1-\sin^2\phi\cos^2\theta}$ (B) $\gamma\sqrt{1+\cos^2\theta\sin^2\phi}$

(C) $\gamma\sqrt{1-\sin^2\theta\cos^2\phi}$ (D) $\gamma\sqrt{1+\cos^2\phi\sin^2\theta}$

Ans. : a

$P(x, y, z), Q(x, y, O); x^2+y^2+z^2=\gamma^2$

$\overline{OQ} = xi + y$

$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

$\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\Rightarrow \sin^2\phi = \frac{x^2+y^2}{x^2+y^2+z^2}$

distance of P from x-axis $\sqrt{y^2+z^2}$

$\Rightarrow \sqrt{\gamma^2-x^2} \Rightarrow \gamma\sqrt{1-\frac{x^2}{\gamma^2}}$

$= \gamma\sqrt{1-\cos^2\theta\sin^2\phi}$

52. The co-ordinates axes are rotated through an angle 135^o. If the coordinates of a point P in the new system are known to be $(4,-3)$, then the coordinates of P in the original system are

(A) $\left(\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$ (B) $\left(\frac{1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$ (C) $\left(\frac{-1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$ (D) $\left(\frac{-1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$

Ans. : d

(d) $P=(x\cos\theta-y\sin\theta,\, x\sin\theta+y\cos\theta)$

$=\left(4.\frac{-1}{\sqrt{2}}+3.\frac{1}{\sqrt{2}},\, 4.\frac{1}{\sqrt{2}}+3.\frac{1}{\sqrt{2}}\right)=\left(\frac{-1}{\sqrt{2}},\,\frac{7}{\sqrt{2}}\right)$

53. Two fixed points are $A(a,0)$ and $B(-a,0)$. If $\angle A-\angle B=\theta$, then the locus of point C of triangle ABC will be

(A) $x^2+y^2+2xy\tan\theta=a^2$ (B) $x^2-y^2+2xy\tan\theta=a^2$

(C) $x^2+y^2+2xy\cot\theta=a^2$ (D) $x^2-y^2+2xy\cot\theta=a^2$

Ans. : d

(d) Given $\angle A-\angle B=\theta \Rightarrow \tan(A-B)=\tan\theta$ .....(i)

In right angled triangle CDA, $\tan A=\frac{k}{a-h}$

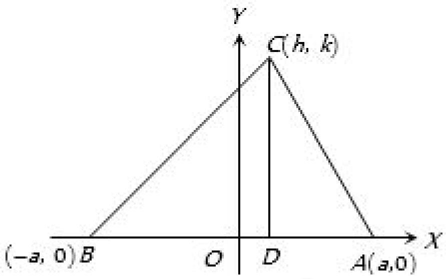
and similarly in triangle CDB, $\tan B=\frac{k}{a+h}$

Also from (i), $\frac{\tan A-\tan B}{1+\tan A\,.\,\tan B}=\tan\theta$

Substituting the values of $\tan A$ and $\tan B$, we get

$h^2-k^2+2hk\cot\theta=a^2$

Hence the locus is $x^2-y^2+2xy\cot\theta=a^2$.



54. Without changing the direction of coordinate axes, origin is transferred to (h,k), so that the linear (one degree) terms in the equation $x^2+y^2-4x+6y-7=0$ are eliminated. Then the point (h,k) is

(A) $(3,2)$ (B) $(-3,2)$ (C) $(2,-3)$ (D) None of these

Ans. : c

(c) Putting $x=x'+h,\ y=y'+k,$ the given equation transforms to

$x'^2+y'^2+x'(2h-4)+y'(2k+6)+h^2+k^2-7=0$

To eliminate linear terms, we should have

$2h-4=0,\ 2k+6=0 \ \Rightarrow\ h=2,\ k=-3$

i.e., $(h,\ k)=(2,\ -3)$.

55. The mid points of three sides of a triangle are $(1,2)$; $(-1,1)$ and $(0,3)$. Area of this triangle will be (in sq. units)⁻

(A) 2 (B) 3 (C) 4 (D) 6

Ans. : d

Area $=4\times\frac{1}{2}\left|\begin{vmatrix}1 & 2 & 1\\ -1 & 1 & 1\\ 0 & 3 & 1\end{vmatrix}\right|=6$

56. Number of values of λ for which the points given by $(\lambda+1,1),(2\lambda+1,3)$ & $(2\lambda+2,2\lambda)$ are collinear, is-

(A) 0 (B) 1 (C) 2 (D) 4

Ans. : c

$A(2+1,1)\quad B(2\lambda+1,3)\quad c(2\lambda+2$

ase said to be cerlineas if Area $(\Delta ABC)=0$

$$=\frac{S}{2}\begin{vmatrix}2 & y & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=0$$

$$\therefore\quad \begin{vmatrix}\lambda+1 & 1 & 1\\ 2\lambda+1 & 3 & 1\\ 2\lambda+2 & 2\lambda & 1\end{vmatrix}=0$$

$\Rightarrow(\dot{x}+1)(3-2\lambda)-1(82+1-2\lambda-2)$

$+1\left(4x^2+2x-6x-6\right)=0$

$\Rightarrow 3\lambda+3-2\left(\lambda^2-2\lambda+1+4x^2-4\lambda-6=0\right.$

$\Rightarrow 2\lambda^2-3\lambda-2=0$

$\Rightarrow 2\lambda^2-4\lambda+\lambda-2=0$

a) $2x(x-2)+1(x-2)=0$

$\Rightarrow \lambda=-1/2$ or 2

" polsible values of

$\lambda \text{are} 2$

57. Area of the triangle formed by points $(102,-4),(105,-2)$ and $(103,-3)$ -

(A) 1 (B) 2 (C) 0.5 (D) 0.25

Ans. : c

Shift origin at $(102,-4)$

58. If the vertices of a triangle be $(0,0)$, $(6,0)$ and $(6,8)$ then its incentre will be

(A) $(2,1)$ (B) $(1,2)$ (C) $(4,2)$ (D) $(2,4)$

Ans. : c

(c) Let $A(0,\ 0),\ B(6,\ 0)$ and $C(6,8)$

Thus $c=AB=6,\ a=BC=8$ and $b=AC=10$

Hence incentre

$=\left(\frac{8\times0+10\times6+6\times6}{8+10+6},\ \frac{8\times0+10\times0+6\times8}{8+10+6}\right)=(4,\ 2)$.

59. Coordinates of the orthocentre of the triangle whose sides are $x=3, y=4$ and $3x+4y=6$ is

(A) $(0,0)$ (B) $(3,0)$ (C) $(0,4)$ (D) $(3,4)$

Ans. : d

(d) Obviously it is a right angled at $(3,4)$. Hence the orthocentre is $(3,4)$

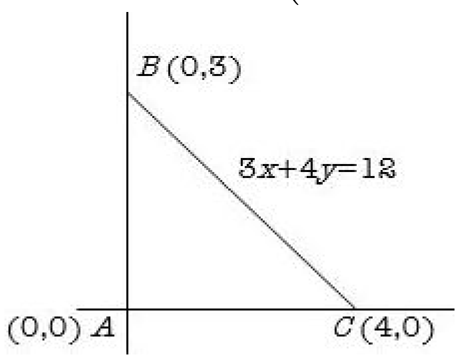
60. The incentre of triangle formed by the lines $x=0,\ y=0$ and $3x+4y=12$ is

(A) $\left(\frac{1}{2},\ \frac{1}{2}\right)$ (B) $(1,1)$ (C) $\left(1,\ \frac{1}{2}\right)$ (D) $\left(\frac{11}{2},1\right)$

Ans. : b

(b)Here $a=BC=5,\ b=AC=4,\ c=AB=3$

Hence incentre is $\left(\frac{0+0+3\times4}{5+4+3},\frac{0+4\times3+0}{5+4+3}\right)$= $(1,1)$.



61. The orthocentre of the triangle formed by $(0,0), (8,0), (46)$ is

(A) $\left(4,\ \frac{8}{3}\right)$ (B) $(3,4)$ (C) $(4,3)$ (D) $(-3,4)$

Ans. : a

(a) Let the vertices of the triangle be $O(0,0)$ $A(8,0)$, $B(4,6)$. The equation of an altitude through O and perpendicular to AB is $y=\frac{2}{3}x$ and the equation of an altitude through $A(8,0)$ and perpendicular to OB is $3y=-2x+16$. The two altitudes intersect at $\left(4,\frac{8}{3}\right)$.

62. The circumcentre of a triangle formed by the line $xy+2x+2y+4=0$ and $x+y+2=0$ is

(A) $(-1,-1)$ (B) $(0,-1)$ (C) $(1,1)$ (D) $(-1,0)$

Ans. : a

(a) $xy+2x+2y+4=0$ ..... (i)

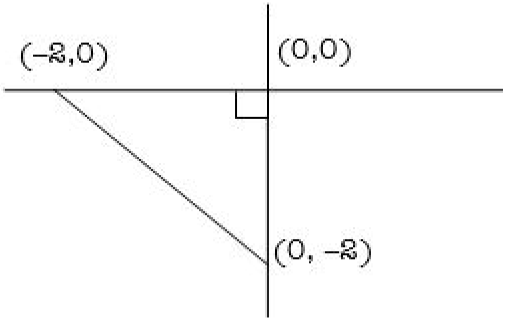
and $x+y+2=0$ ...... (ii)

from (i) and (ii), $xy=0\Rightarrow x=y=0$

Vertices of triangle are $(-2,0)$ $(0,0)$ $(0,-2)$

(In a right angled triangle circumcentre is mid point of hypotenuse)

$(-1,-1)$ is the circumcircle.



63. Orthocentre of the triangle whose vertices are $(0,0)$ $(3,0)$ and $(0,4)$ is

(A) $(0,0)$ (B) $(1,1)$ (C) $(2,2)$ (D) $(3,3)$

Ans. : a

(a) This is a right angled (at origin) triangle therefore orthocentre = $(0,0)$.

64. The incentre of a triangle with vertices $(7,1)$ $(-1,5)$ and $(3+2\sqrt{3},\,3+4\sqrt{3})$ is

(A) $\left(3+\frac{2}{\sqrt{3}},\,3+\frac{4}{\sqrt{3}}\right)$ (B) $\left(1+\frac{2}{3\sqrt{3}},\,1+\frac{4}{3\sqrt{3}}\right)$

(C) $(7,1)$ (D) None of these

Ans. : a

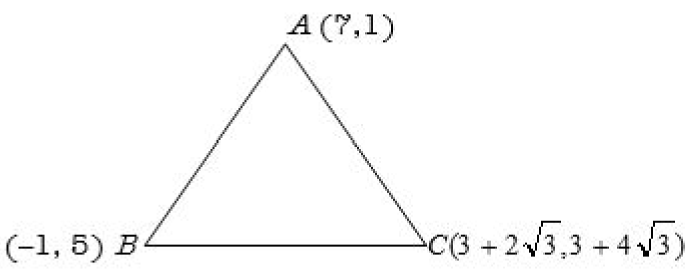
(a) $AB=BC=CA=4\sqrt{5}$

i.e., given triangle is equilateral.

(In centre of a triangle are same as the centriod when triangle is equilateral)

.Hence, incentre = $\left(\frac{7-1+3+2\sqrt{3}}{3},\frac{1+5+3+4\sqrt{3}}{3}\right)$

$=\left(3+\frac{2}{\sqrt{3}},3+\frac{4}{\sqrt{3}}\right)$.



65. If the points $(x+1,\,2)$, $(1,x+2)$, $\left(\frac{1}{x+1},\frac{2}{x+1}\right)$ are collinear, then x is

(A) 4 (B) 0 (C) -4 (D) (b) and (c) both

Ans. : d

(b) Let $A\equiv(x+1,\,2),\ B\equiv(1,\,x+2),\ C\equiv\left(\frac{1}{x+1},\ \frac{2}{x+1}\right)$

then A, B, C are collinear if area of $\Delta ABC=0$

$$\Rightarrow \begin{vmatrix} x+1 & 2 & 1 \\ 1 & x+2 & 1 \\ \frac{1}{x+1} & \frac{2}{x+1} & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & -x & 0 \\ 1 & x+2 & 1 \\ \frac{1}{x+1} & \frac{2}{x+1} & 1 \end{vmatrix} = 0$$

$(R_1 \to R_1 - R_2)$

$$\Rightarrow \begin{vmatrix} x & 0 & 0 \\ 1 & x+3 & 1 \\ \frac{1}{x+1} & \frac{3}{x+1} & 1 \end{vmatrix} = 0 \;\; (C_2 \to C_2 + C_1)$$

$\Rightarrow x\left(x+3-\frac{3}{x+1}\right)=0$

$\Rightarrow x(x^2+3+4x-3)=0$

$\Rightarrow x^2(x+4)=0$

$\Rightarrow x=0,\ -4.$

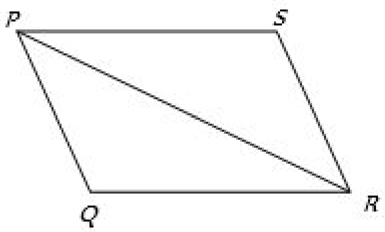
66. $P(2,1), Q(4,-1), R(3,2)$ are the vertices of triangle and if through P and R lines parallel to opposite sides are drawn to intersect in S, then the area of $PQRS$ is

(A) 6 (B) 4 (C) 8 (D) 12

Ans. : b

(b) Since $PQRS$ is a parallelogram with an area which is twice the area of ΔPQR.

Therefore, area $PQRS = 2.\frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 4 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 4.$

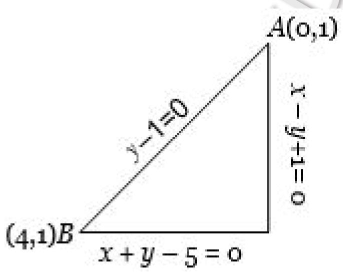


67. The equations of the sides of a triangle are $x+y-5=0$; $x-y+1=0$ and $y-1=0$, then the coordinates of the circumcentre are

(A) $(2,1)$ (B) $(1,2)$ (C) $(2,-2)$ (D) $(1,-2)$

Ans. : a

(a) Since the triangle is right angled so the circumcentre will be the middle point of hypotenuse i.e. $(2,1)$.



68. The incentre of the triangle formed by $(0,0)$, $(5,12)$, $(16,12)$ is

(A) $(7,9)$ (B) $(9,7)$ (C) $(-9,7)$ (D) $(-7,9)$

Ans. : a

(a) Obviously $a=11,\ b=20,\ c=13$.

Hence incentre is

$\left(\frac{11\times0+20\times5+13\times16}{11+20+13},\ \frac{11\times0+20\times12+13\times12}{11+20+13}\right)$ i.e. $(7,9)$.

69. Circumcenter of the triangle formed by the line $y=x,\ \ y=2x$ and $y=3x+4$ is

(A) $(6,8)$ (B) $(6,-8)$ (C) $(3,4)$ (D) $(-3,-4)$

Ans. : b

(b) $x^2+y^2=(x+4)^2+(y+8)^2 \ \Rightarrow\ 8x+16y+80=0$

and $x^2+y^2=(x+2)^2+(y+2)^2 \ \Rightarrow\ 4x+4y+8=0$

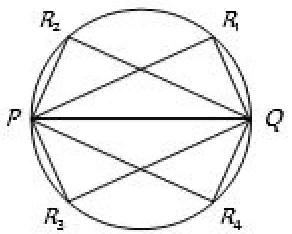
On simplification, we get $y=-8$ and $x=6$.

70. $P(3,1)$, $Q(6,5)$ and $R(x,y)$ are three points such that the angle PRQ is a right angle and the area of the $\Delta RPQ=5$, then the number of such points R is

(A) 0 (B) 1 (C) 2 (D) 4

Ans. : d

(d) In the following figure, the four possible points are seen.



71. In a $\triangle ABC$, the angle bisector BD of $\angle B$ intersects AC in D. Suppose $BC=2, CD=1$ and $BD=\frac{3}{\sqrt{2}}$. The perimeter of the $\triangle ABC$ is

(A) $\frac{17}{2}$ (B) $\frac{15}{2}$ (C) $\frac{17}{4}$ (D) $\frac{15}{4}$

Ans. : b

(b)

We have, $\cos\frac{B}{2}=\frac{\frac{9}{2}+4-1}{6\sqrt{2}}=\frac{5}{4\sqrt{2}}$

$\therefore$ Length of angle bisector,

$BD=\frac{2ac}{a+c}\cos\frac{B}{2}$

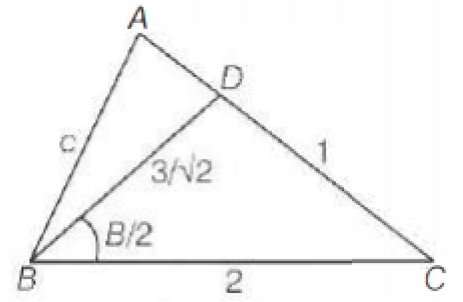
$\frac{3}{\sqrt{2}}=\left(\frac{4c}{e+2}\right)\cdot\frac{5}{4\sqrt{2}}$

$c=3$

We know that, $\frac{AB}{BC}=\frac{AD}{CD}$

$AD=\frac{3}{2}$

$\therefore$ Perimeter of $\triangle ABC = 1 + \frac{3}{2} + 3 + 2 = \frac{15}{2}$



72. In a triangle $ABC, \angle BAC = 90^\circ; AD$ is the altitude from A on to BC. Draw DE perpendicular to AC and DF perpendicular to AB. Suppose $AB = 15$ and $BC = 25$. Then the length of EF is

(A) 12 (B) 10 (C) $5\sqrt{3}$ (D) $5\sqrt{5}$

Ans. : a

(a)

It is given that in triangle ABC, $\angle BAC = 90^\circ, AD$ is the altitude from A on to BC.

Since, $AB = 15$ and $BC = 25$

$\therefore \quad AC = \sqrt{BC^2 - AB^2} = \sqrt{625 - 225}$

$= \sqrt{400} = 20$

Now, since area of $\triangle ABC = \frac{1}{2}(BC)(AD)$

$= \frac{1}{2}(AB)(AC)$

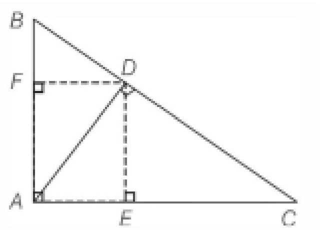
$\Rightarrow \frac{1}{2}(BC)(AD) = \frac{1}{2} \times 15 \times 20$

$\Rightarrow \quad 25 \times AD = 300$

$\Rightarrow \quad AD = 12$

$\because AEDF$ is a rectangle, then

$EF = AD = 12$



73. Let ABC be an equilateral triangle with side length a. Let R and r denote the radii of the circumcircle and the incircle of triangle ABC respectively. Then, as a function of a, the ratio $\frac{R}{r}$

(A) strictly increases

(B) strictly decreases

(C) remains constant

(D) strictly increases for $a < 1$ and strictly decrease for $a > 1$

Ans. : c

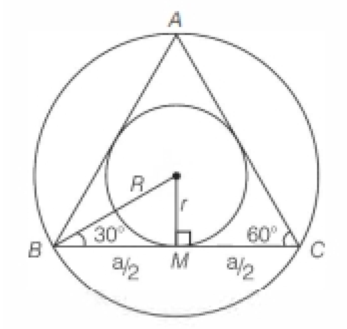
(c)

For an equilateral triangle ABC having side length a. If R and r are radii of the circumcircle and the incircle of triangle ABC respectively, then

$R = \frac{a}{2}\sec 30^\circ = \frac{a}{2}\left(\frac{2}{\sqrt{3}}\right) = \frac{a}{\sqrt{3}}$

and $r = \frac{a}{2}\tan 30^\circ = \frac{a}{2} \times \frac{1}{\sqrt{3}} = \frac{a}{2\sqrt{3}}$

$\therefore \frac{R}{r} = \frac{\frac{a}{\sqrt{3}}}{\frac{a}{2\sqrt{3}}} = 2$, which is independent of a and it is constant.



74. If coordinates of the points A and B are $(2,4)$ and $(4,2)$ respectively and point M is such that $A-M-B$ also $AB = 3\,AM$, then the coordinates of M are

(A) $\left(\frac{8}{3}, \frac{10}{3}\right)$ (B) $\left(\frac{10}{3}, \frac{14}{4}\right)$ (C) $\left(\frac{10}{3}, \frac{6}{3}\right)$ (D) $\left(\frac{13}{4}, \frac{10}{4}\right)$

Ans. : a

(a) Since $AB = 3AM$

$\Rightarrow\ AM + MB = 3AM\ \Rightarrow\ MB = 2AM$

Hence ratio $AM\ :\ MB = 1\ :\ 2$

Therefore the point $M\ \left(\frac{8}{3},\ \frac{10}{3}\right)$.

75. What is the equation of the locus of a point which moves such that 4 times its distance from the x-axis is the square of its distance from the origin

(A) $x^2 + y^2 - 4y = 0$ (B) $x^2 + y^2 - 4|y| = 0$ (C) $x^2 + y^2 - 4x = 0$ (D) $x^2 + y^2 - 4|x| = 0$

Ans. : b

(b) Let the required point be (x_1, y_1).

Then, according to question, $4|y_1| = (x_1^2 + y_1^2)$

==> $x_1^2 + y_1^2 - 4\,|y_1| = 0$

Replace (x_1, y_1) from (x, y), then $x^2 + y^2 - 4y = 0$ is the required locus.

76. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of c is

(A) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (B) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

(C) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (D) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

Ans. : d

(d) Let (h, k) be the point on the locus, then by the given conditions $(h-a_1)^2+(k-b_1)^2=(h-a_2)^2+(k-b_2)^2$

$\Rightarrow\ 2h(a_1-a_2)+2k(b_1-b_2)+a_2^2+b_2^2-a_1^2-b_1^2=0$

$\Rightarrow\ h(a_1-a_2)+k(b_1-b_2)+\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)=0$ ....(i)

Also, since (h,k) lies on the given locus, therefore

$(a_1-a_2)h+(b_1-b_2)k+c=0$ .....(ii)

Comparing (i) and (ii), we get

$c=\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)$.

77. The locus of the moving point P, such that $2PA=3PB$ where A is $(0,0)$ and B is $(4,-3)$, is

(A) $5x^2-5y^2-72x+54y+225=0$
(B) $5x^2-5y^2+72x+54y+225=0$
(C) $5x^2+5y^2+72x+54y+225=0$
(D) $5x^2+5y^2-72x+54y+225=0$

Ans. : d

(d) Let $P(h,\ k)$. Given $2PA=3PB\ \Rightarrow\ 4PA^2=9PB^2$

$\Rightarrow\ 4(h^2+k^2)=9[(h-4)^2+(k+3)^2]$

$\Rightarrow\ 5h^2+5k^2-72h+54k+225=0$

Hence the locus of point P is given by

$5x^2+5y^2-72x+54y+225=0$.

78. The equation of the locus of all points equidistant from the point $(4,2)$ and the x-axis, is

(A) $x^2+8x+4y-20=0$
(B) $x^2-8x-4y+20=0$
(C) $y^2-4y-8x+20=0$
(D) None of these

Ans. : b

(b) $(x-4)^2+(y-2)^2=y^2$

$\Rightarrow\ x^2-8x-4y+20=0$.

79. A point moves in such a way that the sum of square of its distance from the points $A(2,0)$ and $B(-2,0)$ is always equal to the square of the distance between A and B. The locus of the point is

(A) $x^2+y^2-2=0$ (B) $x^2+y^2+2=0$ (C) $x^2+y^2+4=0$ (D) $x^2+y^2-4=0$

Ans. : d

(d) $(x-2)^2+y^2+(x+2)^2+y^2=16$

$\Rightarrow\ x^2+y^2=4$.

80. If the coordinates of a point be given by the equation $x=a(1-\cos\theta),\ y=a\sin\theta$, then the locus of the point will be

(A) A straight line (B) A circle (C) A parabola (D) An ellipse

Ans. : b

(b) On eliminating θ, we get the required locus.

Since $x = a(1-\cos\theta) \Rightarrow \; x - a = -a\cos\theta$ .....(i)

and $y = a\sin\theta$ .....(ii)

Now adding the squares of (i) and (ii), we get

$x^2 + y^2 - 2ax = 0$, which is equation of a circle.

81. The locus of a point P which moves in such a way that the segment OP, where O is the origin, has slope $\sqrt{3}$ is

(A) $x - \sqrt{3}y = 0$ (B) $x + \sqrt{3}y = 0$ (C) $\sqrt{3}x + y = 0$ (D) $\sqrt{3}x - y = 0$

Ans. : d

(d) Slope is given by $\frac{dy}{dx} = \sqrt{3} \;\Rightarrow\; \int dy = \sqrt{3}\int dx$

$\Rightarrow \; \sqrt{3}x - y + c = 0$

This passes through $(0,0)$, so $c = 0$

Hence the required locus is $\sqrt{3}x - y = 0$.

82. The points $(1,1)$, $(0,\sec^2\theta)$, $(\operatorname{cosec}^2\theta, 0)$ are collinear for

(A) $\theta = \frac{n\pi}{2}$ (B) $\theta \neq \frac{n\pi}{2}$ (C) $\theta = n\pi$ (D) None of these

Ans. : b

(b) The given points are collinear, if

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & \sec^2\theta & 1 \\ \operatorname{cosec}^2\theta & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow \; 1(\sec^2\theta) + 1(\operatorname{cosec}^2\theta) - 1(\operatorname{cosec}^2\theta.\sec^2\theta) = 0$

$\Rightarrow \; \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} - \frac{1}{\sin^2\theta\cos^2\theta} = 0$

$\Rightarrow \; \frac{1}{\cos^2\theta\sin^2\theta} - \frac{1}{\sin^2\theta\cos^2\theta} = 0 \;\Rightarrow\; 0 = 0$

Therefore, the points are collinear for all values of θ, except only $\theta = \frac{n\pi}{2}$ because at $\theta = \frac{n\pi}{2}$, $\sec^2\theta = \infty$.

83. If $A(at^2, 2at)$, $B(a/t^2, -2a/t)$ and $C(a, 0)$, then $2a$ is equal to

(A) $A.M.$ of CA and CB (B) $G.M.$ of CA and CB

(C) $H.M.$ of CA and CB (D) None of these

Ans. : c

(c) $CA = \sqrt{(at^2 - a)^2 + (2at)^2} = a\sqrt{(t^2-1)^2 + 4t^2}$

$= a\sqrt{(t^2 + 1 + 2t^2)} = a(1+t^2)$

$CB = \sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{-2a}{t}\right)^2} = a\left(1 + \frac{1}{t^2}\right)$

$H.M.$ of CA and $CB = \frac{2a^2(1+t^2)\left(1+\frac{1}{t^2}\right)}{a\left[1+t^2+1+\frac{1}{t^2}\right]} = 2a.$

$\left[\because \text{ H.M. of } x \text{and } y = \frac{2xy}{x+y}\right]$

84. Point of intersection of the diagonals of square is at origin and coordinate axis are drawn along the diagonals. If the side is of length a, then one which is not the vertex of square is

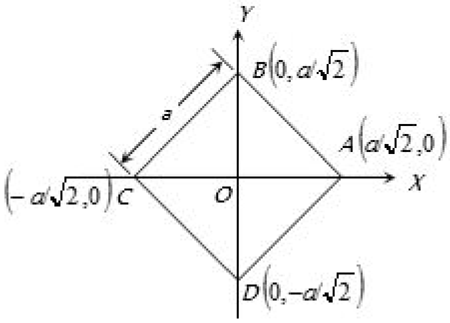
(A) $(a\sqrt{2}, 0)$ (B) $\left(0, \frac{a}{\sqrt{2}}\right)$ (C) $\left(\frac{a}{\sqrt{2}}, 0\right)$ (D) $\left(-\frac{a}{\sqrt{2}}, 0\right)$

Ans. : a

(a) Obviously, from right angled triangle BOA

$OA = OB = \frac{a}{\sqrt{2}}$

Hence the vertex $(a\sqrt{2},\ 0)$ is not the vertex of square.



85. Two vertices of a triangle are $(4, -3)$ and $(-2, 5)$. If the orthocentre of the triangle is at $(1, 2)$, then the third vertex is

(A) $(-33, -26)$ (B) $(33, 26)$ (C) $(26, 33)$ (D) None of these

Ans. : b

(b) Let third vertex be (h, k). Now slope of AD is $\frac{k-2}{h-1}$,

Slope of BC is $\frac{5+3}{-2-4} = \frac{-4}{3}$

Slope of BE is $\frac{-3-2}{4-1} = \frac{-5}{3}$

And slope of AC is $\frac{k-5}{h+2}$

Since $AD \perp BC \Rightarrow \frac{k-2}{h-1} \times \frac{-4}{3} = -1$

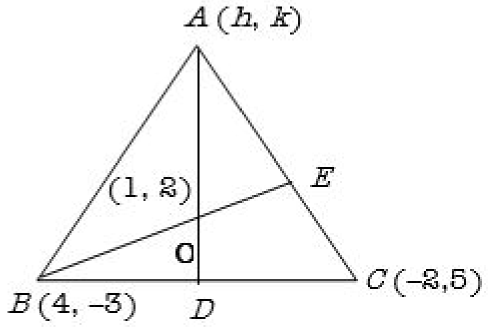
$\Rightarrow 3h - 4k + 5 = 0$ ......(i)

Again Since $BE \perp AC \Rightarrow -\frac{5}{3} \times \frac{k-5}{h+2} = -1$

==> $3h - 5k + 31 = 0$ .....(ii)

on solving (i) and (ii) we get $h = 33, k = 26$

Hence the third vertex is $(33,26)$.



86. The coordinates of the points A, B, C are (x_1, y_1), (x_2, y_2), (x_3, y_3) and D divides the line AB in the ratio $l : k$. If P divides the line DC in the ratio $m : k + l$, then the coordinates of P are

(A) $\left(\frac{kx_1+lx_2+mx_3}{k+l+m}, \frac{ky_1+ly_2+my_3}{k+l+m}\right)$

(B) $\left(\frac{lx_1+mx_2+kx_3}{l+m+k}, \frac{ly_1+my_2+ky_3}{l+m+k}\right)$

(C) $\left(\frac{mx_1+kx_2+lx_3}{m+k+l}, \frac{my_1+ky_2+ly_3}{m+k+l}\right)$

(D) None of these

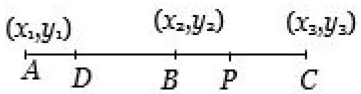
Ans. : a

(a) Coordinates of D will be $\left(\frac{lx_2+kx_1}{l+k}, \frac{ly_2+ky_1}{l+k}\right)$

Now again, DC is divided by P in $m : k + l$.

Then the coordinates of P will be given by

$\left(\frac{mx_3+lx_2+kx_1}{k+l+m}, \frac{my_3+ly_2+ky_1}{k+l+m}\right)$.



87. Let $A(h, k)$, $B(1,1)$ and $C(2,1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1 square unit, then the set of values which $'k'$ can take is given by

(A) $-1, 3$

(B) $-3, -2$

(C) $1, 3$

(D) $0, 2$

Ans. : a

Now, $AB = \sqrt{(1-h)^2 + (1-k)^2}$

$BC = \sqrt{(2-1)^2 + (1-1)^2}$

and $\quad CA = \sqrt{(h-2)^2 + (k-1)^2}$

From Pythagoras theorem, $AC^2 = AB^2 + BC^2$

$\Rightarrow 4 + h^2 - 4h + k^2 + 1 - 2k$

$= h^2 + 1 - 2h + k^2 + 1 - 2k + 1$

$\Rightarrow \quad 5 - 4h = 3 - 2h$

$h = 1$

Now, given that area of triangle is 1

Then, area $(\Delta ABC) = \frac{1}{2} \times AB \times BC$

$\Rightarrow 1 = \frac{1}{2} \times \sqrt{(1-h)^2 + (1-k)^2} \times 1$

$\Rightarrow 2 = \sqrt{(1-n)^2 + (1-k)^2}$

$\Rightarrow 2 = \sqrt{(k-1)^2}$ [from Eq. (i)]

$\Rightarrow 4 = k^2 + 1 - 2k$

$\Rightarrow k^2 - 2k - 3 = 0$

$\Rightarrow (k-3)(k+1) = 0$

$\therefore \quad k = -1, 3$

Thus, the set of values of k is $\{-1, 3\}$

88. Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line

(A) $3x - 2y = 3$ (B) $2x - 3y = 7$ (C) $3x + 2y = 5$ (D) $2x + 3y = 9$

Ans. : d

Let the third vertex be (x_1, y_1) then

Centroid $(G) \equiv \left(\frac{x_1+2-2}{3}, \frac{y_1-3+1}{3}\right)$

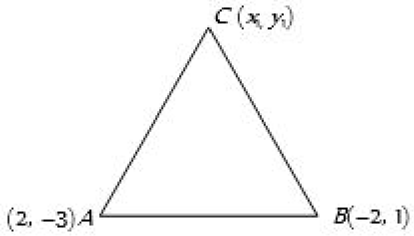
i.e., $G\left(\frac{x_1}{3}, \frac{y_1-2}{3}\right)$

Given, centroid of triangle moves on the line

$2x + 3y = 1$

$\therefore 2\left(\frac{x_1}{3}\right) + 3\left(\frac{y_1-2}{3}\right) = 1$ i.e., $2x_1 + 3y_1 = 9$

Locus of (x_1, y_1) is $2x + 3y = 9$.



89. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$, where t is a parameter; is

(A) $(3x-1)^2 + (3y)^2 = a^2 - b^2$ (B) $(3x-1)^2 + (3y)^2 = a^2 + b^2$

(C) $(3x+1)^2 + (3y)^2 = a^2 + b^2$ (D) $(3x+1)^2 + (3y)^2 = a^2 - b^2$

Ans. : b

(b) $3h = a\cos t + b\sin t + 1,\ 3k = a\sin t - b\cos t$

$a^2 + b^2 = (3h-1)^2 + (3k)^2$

$(3x-1)^2 + (3y)^2 = a^2 + b^2$.

90. The locus of the mid-point of the distance between the axes of the variable line $x\cos\alpha + y\sin\alpha = p$, where p is constant, is

(A) $x^2 + y^2 = 4p^2$ (B) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ (C) $x^2 + y^2 = \frac{4}{p^2}$ (D) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$

Ans. : b

(b) The straight line $x\cos\alpha + y\sin\alpha = p$ meets the x-axis at the point $A\left(\frac{p}{\cos\alpha}, 0\right)$ and the y-axis at the point $B\left(0,\ \frac{p}{\sin\alpha}\right)$. Let (h, k) be the coordinates of the middle point of the line segment AB.

Then, $h = \frac{p}{2\ \cos\alpha}$ and $k = \frac{p}{2\ \sin\alpha}$

$\Rightarrow\ \cos\alpha = \frac{p}{2h}$ and $\sin\alpha = \frac{p}{2k}$

$\Rightarrow\ \sin^2\alpha + \cos^2\alpha = \frac{p^2}{4h^2} + \frac{p^2}{4k^2} = 1$

Hence locus of the point (h, k) is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

91. To remove xy term from the second degree equation $5x^2 + 8xy + 5y^2 + 3x + 2y + 5 = 0$, the coordinates axes are rotated through an angle θ, then θ equals:-

(A) $\pi/2$ (B) $\pi/4$ (C) $3\pi/8$ (D) $\pi/8$

Ans. : b

$*Ax^2 + 2hxy + By^2 + 2gx + 2fy + c = 0$

To eliminate xy tem θ'

$\Rightarrow|\ \tan 2\theta = \frac{2h}{A-\beta}$

$2\theta = \tan^{-1}\left(\frac{2h}{A-B}\right)$

$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2n}{A-B}\right)$

$= \frac{1}{2}\tan^{-1}\left(\frac{8}{5-5}\right)$

$\theta = \frac{1}{2}\tan^{-1}(\infty)$

$\theta = \frac{1}{2} \times \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

92. If the axes be rotated through an angle $\frac{\pi}{3}$ in the clockwise direction with respect to $(0,0)$ the point $(4,2)$ in the new system was formally-

(A) $(2+\sqrt{3}, -2\sqrt{3}-1)$ (B) $(-2\sqrt{3}+1, 2+\sqrt{3})$

(C) $(2+\sqrt{3}, -2\sqrt{3}+1)$ (D) $(2-\sqrt{3}, -2\sqrt{3}-1)$

Ans. : c

	x	y
4	$1/2$	$-\sqrt{3}$ $/2$
2	$\sqrt{3}/2$	$1/2$

$x = 4\left(\frac{1}{2}\right) + 2\left(\frac{\sqrt{3}}{2}\right) = 2 + \sqrt{3}$

$y = 4\left(-\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{2}\right) = -2\sqrt{3} + 1$

93. A point moves in the $x-y$ plane such that the sum of its distances from two mutually perpendicular lines is always equal to 3. The area enclosed by the locus of the point is- .... unit^2

(A) 18 (B) 4.5 (C) 9 (D) None of these

Ans. : a

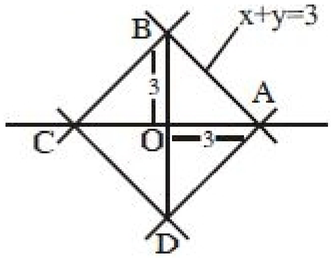
$|x|+|y|=3$

shows square

and area of square $ABCD$

$=4\,(are\ of\ \Delta OAB)$

$=4\times\frac{1}{2}\times3\times3=18$ unit ^{2}



94. Let $A\,(2,3)$ and $B\,(-4,5)$ are two fixed points. A point P moves in such a way that $\Delta PAB=12\,sq.units$, then its locus is :-

(A) $x^2+6xy+9y^2+22x+66y-23=0$

(B) $x^2+6xy+9y^2+22x+66y+23=0$

(C) $x^2+6xy+9y^2-22x-66y-23=0$

(D) none of these

Ans. : c

Let, $P=(x,y)$

$A=(2,3)$ and $B=(-4,5)$

$\because Ar.(\triangle PAB)=12$ sq. units

$\Longrightarrow \frac{1}{2}|x(3-5)+2(5-y)-4(y-3)|=12$

$\Longrightarrow |-2x+10-2y-4y+12|=24$

$\Longrightarrow |-2x-6y+22|=24$

$\Longrightarrow 2x+6y-22=\pm24$

$\Rightarrow x+3y-11=\pm12$

$\Longrightarrow x+3y-23=0$ or $x+3y+1=0$

Hence, the locus is $(x+3y-23)(x+3y+1)=0$

$\Longrightarrow x^2+6xy+9y^2-22x-66y-23=0$

95. Area of the triangle formed by the lines $y^2-9xy+18x^2=0$ and $y=9$, is .... sq. unit

(A) 27 (B) 13.5 (C) 6.75 (D) 3.375

Ans. : c

Put $y=9$ in given pair of straight lines

$18x^2-81x+81=0$

$\Rightarrow \quad x=3,\frac{3}{2}$

So, we have three vertices $(0,0),(3,9)$

$\text{and}\left(\frac{3}{2},9\right)$

Hence $\Delta=\frac{27}{4}$

96. The area enclosed by the graphs of $|x+y|=2$ and $|x|=1$ is

(A) 2 (B) 4 (C) 6 (D) 8

Ans. : d

$|x+y|=2$

$\Rightarrow x+y=\pm 2$

$|x|=1$

$\Rightarrow x=\pm 1$

From the fig, attached, the resultant fig., is a parallelogram having vertices (-1,3), (1,1),(1,3)

and (-1,-1)

Therefore,

$BP=2$ units

$BC=4$ units

Therefore,Area of parallelogram $ABCD=BP\times BC=2\times 4=8$ sq. units.

Hence the correct answer is $(D)8$.

97. If α,β,γ are the real roots of the equation $x^3-3px^2+3qx-1=0$, then the centroid of the triangle whose vertices are $(\alpha,\frac{1}{\alpha}),(\beta,\frac{1}{\beta})$ and $(\gamma,\frac{1}{\gamma})$

(A) $p,-q$ (B) $(-p,q)$ (C) (p,q) (D) $(\frac{p}{2},\frac{q}{2})$

Ans. : c

The centroid of the given triangle is the point $\left(\frac{\alpha+\beta+\gamma}{3},\frac{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}}{3}\right)=$

$\left(\frac{3p}{3},\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{3\alpha\beta\gamma}\right)=(p,q)$

$[\because \alpha+\beta+\gamma=3p,\alpha\beta+\beta\gamma+\gamma\alpha=3q,\alpha\beta+\beta\gamma+\gamma\alpha=3q,\alpha\beta\gamma=1]$

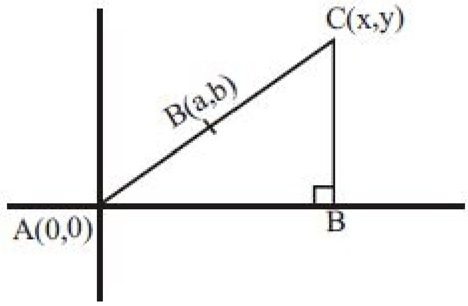
98. The orthocentre of a ΔABC is $'B'$ and circumcentre is $S(a,b)$. If A is origin then coordinate of C is-

(A) $(2a,2b)$ (B) $(\frac{a}{2},\frac{b}{2})$ (C) $(\sqrt{a^2+b^2},0)$ (D) None of these

Ans. : a

$\frac{\mathrm{x}+0}{2}=\mathrm{a}\Rightarrow \mathrm{x}=2\mathrm{a} \qquad c\,(2a,2b)$

$\frac{y+0}{2}=b \Rightarrow y=2b$



99. The coordinates of the foot of the perpendiculars from the vertices of a triangle on the opposite sides are $(20,25),(8,16)$ and $(8,9)$. The orthocentre of the triangle lies at the point-

(A) $(5,10)$ (B) $(15,30)$ (C) $(10,15)$ (D) $(50,-5)$

Ans. : c

Let ABC be the triangle and D,E,F represent the given points respectively, then DEF is the pedal triangle. From geometry we know that orthocentre of the triangle ABC is the incentre of the pedal triangle DEF. If $O(h,k)$ denotes the orthocentre of the triangle ABC, then from ΔDEF.

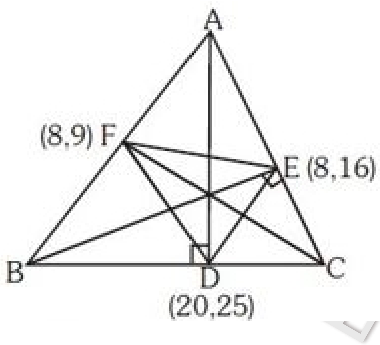
$DE=\sqrt{(20-8)^2+(25-16)^2}=\sqrt{12^2+9^2}=15$

$EF=7$ and $FD=20$

so that

$h=\frac{20\times8+7\times20+15\times8}{7+15+20}=\frac{160+140+120}{42}=10$

and $k=\frac{20\times16+7\times25+15\times9}{7+15+20}=15$



100. If Δ_1 is the area of the triangle formed by the centroid and two vertices of a triangle, Δ_2 is the area of the triangle formed by the mid-points of the sides of the same triangle, then $\Delta_1:\Delta_2=$

(A) $3:4$ (B) $4:1$ (C) $4:3$ (D) $2:1$

Ans. : c

Let $A(x_1,y_1), B(x_2,y_2)$ and $C(x_3,y_3)$ be the vertices of a ΔABC, and let G be its centroid. Then, $\Delta_1=$ Area of ΔGBC

$\Rightarrow \Delta_1=\frac{\Delta}{3}$, where Δ is the area of ΔABC

$\Delta_2 =$ Area of triangle formed by the mid-points of the sides

$\Rightarrow \Delta_2 = \frac{1}{4}\Delta$

$\therefore \Delta_1 : \Delta_2 = 4:3$

101. Number of straight lines that can be drawn from $(2,5)$ which make a triangle of area $24sq.$ units with the coordinate axes is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. : d

Without loss of generality, let us draw lines from $(2,5)$ then the line which has minimum area is the one where $(2,5)$ is the mid-point

$\Lambda_{\min} = \frac{1}{2} \times 4 \times 10 = 20$

$\therefore$ there will be two such lines in the I^{st} quadrant one line in II quadrant and one line in IV quadrant $\Rightarrow$ Total lines $= 4$

102. If the line $y = \sqrt{3}x$ cuts the curve $x^4 + ax^2y + bxy + cx + dy + 6 = 0$ at A , B , C and D , then value of $OA.OB.OC.OD$ is, (where O is origin)

(A) $a+b+c$ (B) $2c^2d$ (C) 96 (D) 6

Ans. : c

Coordinates of any points lying on the line

$y = \sqrt{3}x$ will be $\left(\frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$

If the given line intersects the curve

$\mathrm{x}^4 + \mathrm{ax}^2\mathrm{y} + \mathrm{bxy} + \mathrm{cx} + \mathrm{dy} + 6 = 0,$ then

$\frac{r^4}{16} + a\frac{r^3\sqrt{3}}{8} + \frac{br^2\sqrt{3}}{4} + \frac{cr}{2} + \frac{dr\sqrt{3}}{2} + 6 = 0$

$r^4 + r^3 2a\sqrt{3} + r^2 4b\sqrt{3} + r8(c + d\sqrt{3}) + 96 = 0$

$\therefore \quad \mathrm{r_1r_2r_3r_4} = 96$

hence $\mathrm{OA \cdot OB \cdot OC \cdot OD} = 96$

103. An insect is resting on the graph paper at a point $A(3,2)$. Now it starts moving towards west direction and covers a distance of $4\,units$ and then it turns towards south and covered a distance of $3\,units$ and reaches at point B then the polar co-ordinates of point B will be :-

(A) $\left(6\sqrt{2}, \frac{\pi}{4}\right)$ (B) $\left(\sqrt{2}, \frac{3\pi}{4}\right)$ (C) $\left(\sqrt{2}, \frac{-3\pi}{4}\right)$ (D) None of these

Ans. : (C) $\left(\sqrt{2}, \frac{-3\pi}{4}\right)$

*** Answer the following questions in one sentence. [1 Marks Each]** **[12]**

104. Name the octants in which the following points lie:
(1, 2, 3), (4, -2, 3), (4, -2, -5), (4, 2, -5), (-4, 2, -5), (-4, 2, 5), (-3, -1, 6), (-2, -4, -7)

Ans. :

105. Find the distance between (-1, 3, -4) and (1, -3, 4) pairs of points.

Ans. : Let A(-1, 3, -4) and B(1, -3, 4) be two points. Then

$AB = \sqrt{[1-(-1)]^2 + (-3-3)^2 + [4-(-4)]^2}$ [using distance formula]

$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$

106. Find the distance between (2, -1, 3) and (-2, 1, 3) pairs of points.

Ans. : Let A(2, -1, 3) and B(-2, 1, 3) be two points. Then

$AB = \sqrt{(-2-2)^2 + [1-(-1)]^2 + (3-3)^2}$ [using distance formula]

$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$

$= \sqrt{16+4+0} = \sqrt{20} = 2\sqrt{5}$ units

107. Find the octant in which the points (–3, 1, 2) and (–3, 1, – 2) lie.

Ans. : The point (–3,1, 2) lies in the 2nd octant
and The point (–3, 1, – 2) lies in octant VI.

108. Find the coordinates of the point which divides the line segment joining the points (1, –2, 3) and (3, 4, –5) in the ratio 2 : 3 internally.

Ans. : Let P (x, y, z) be the point which divides the line segment joining A(1, – 2, 3) and B (3, 4, –5) internally in the ratio 2 : 3. Therefore

$x = \frac{2(3)+3(1)}{2+3} = \frac{9}{5}, y = \frac{2(4)+3(-2)}{2+3} = \frac{2}{5}, z = \frac{2(-5)+3(3)}{2+3} = \frac{-1}{5}$ [using section formula]

Thus, the required point is $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$

109. Find the coordinates of a point equidistant from the origin and points A(a, 0, 0), B(0, b, 0) and C(0, 0, c).

Ans. : Let the point be P(x, y, z).
Now, PO = PA

$\sqrt{(0-\mathrm{x})^2 + (0-\mathrm{y})^2 + (0-\mathrm{z})^2} = \sqrt{(\mathrm{a}-\mathrm{x})^2 + (0-\mathrm{y})^2 + (0-\mathrm{z})^2}$

$\Rightarrow \mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2 = \mathrm{a}^2 - 2\mathrm{ax} + \mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2$

$\Rightarrow 0 = \mathrm{a}^2 - 2\mathrm{ax}$

$\Rightarrow \mathrm{x} = \frac{\mathrm{a}}{2}$

Also, PO = PB

$\sqrt{(0-\mathrm{x})^2 + (0-\mathrm{y})^2 + (0-\mathrm{z})^2} = \sqrt{(0-\mathrm{x})^2 + (\mathrm{b}-\mathrm{y})^2 + (0-\mathrm{z})^2}$

$\Rightarrow \mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2 = \mathrm{x}^2 - 2\mathrm{by} + \mathrm{y}^2 + \mathrm{z}^2$

$\Rightarrow 0 = \mathrm{b}^2 - 2\mathrm{by}$

$\Rightarrow \mathrm{y} = \frac{\mathrm{b}}{2}$

Again, PO = PC

$\sqrt{(0-\mathrm{x})^2 + (0-\mathrm{y})^2 + (0-\mathrm{z})^2} = \sqrt{(0-\mathrm{x})^2 + (\mathrm{b}-\mathrm{y})^2 + (\mathrm{c}-\mathrm{z})^2}$

$\Rightarrow \mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2 = \mathrm{x}^2 + \mathrm{y}^2 + \mathrm{c}^2 - 2\mathrm{cz} + \mathrm{z}^2$

$\Rightarrow 0 = \mathrm{c}^2 - 2\mathrm{cz}$

$\Rightarrow \mathrm{z}=\frac{\mathrm{c}}{2}$

Hence, the coordinates of the point is $\left(\frac{\mathrm{a}}{2}, \frac{\mathrm{b}}{2}, \frac{\mathrm{c}}{2}\right)$

110. Find the image of:
(- 2, 3, 4) in the yz-plane.

Ans. : (2, 3, 4)

111. If the origin is the centroid of a triangle ABC having vertices A(a, 1, 3), B(-2, b -5) and C(4, 7, c) find the values of a, b, c.

Ans. : We have A(a, 1, 3), B(-2, b, -5) and C(4, 7, c)
Now,

$\frac{\mathrm{a}-2+4}{3}=0,\ \frac{1+\mathrm{b}+7}{3}=0 \text{ and } \frac{3-5+\mathrm{c}}{3}=0$

$\Rightarrow \mathrm{a}=-2,\ \mathrm{b}=-8 \text{ and } \mathrm{c}=2$

112. Determine the point on yz-plane which is equidistant from points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

Ans. : The coordiante of x point on yz-plane is 0
Let the point be P(0, y, z).
Now, PA = PB

$\sqrt{(2-0)^2+(0-\mathrm{y})^2+(3-\mathrm{z})^2}=\sqrt{(0-0)^2+(3-\mathrm{y})^2+(2-\mathrm{z})^2}$

$\Rightarrow 4+\mathrm{y}^2+9-6\mathrm{z}+\mathrm{z}^2=9-6\mathrm{y}+\mathrm{y}^2+4-4\mathrm{z}+\mathrm{z}^2$

$\Rightarrow -6\mathrm{z}=-6\mathrm{y}-4\mathrm{z}$

$\Rightarrow 3\mathrm{y}-\mathrm{z}=0 \ldots(\mathrm{i})$

Also, PA = PC

$\sqrt{(2-0)^2+(0-\mathrm{y})^2+(3-\mathrm{z})^2}=\sqrt{(0-0)^2+(0-\mathrm{y})^2+(1-\mathrm{z})^2}$

$\Rightarrow 4+\mathrm{y}^2+9-6\mathrm{z}+\mathrm{z}^2=\mathrm{y}^2+1-2\mathrm{z}+\mathrm{z}^2$

$\Rightarrow 13-6\mathrm{z}=1-2\mathrm{z}$

$\Rightarrow -4\mathrm{z}=-12$

$\Rightarrow \mathrm{z}=3 \ldots(\mathrm{ii})$

Solving (i) and (ii), we get
y = 1
Hence, the coordinates of the point is (0, 1, 3).

113. Find the image of:
(-5, 4, -3) in the xz-plane.

Ans. : (-5, -4, -3)

114. Find the ratio in which the line segment joining the points (2, 4, 5) and (3, −5, 4) is divided by the yz-plane.

Ans. : Let the yz-plane divide the line sgement joining the points (2, 4, 5) and (3, −5, 4) in m : 1.
Now, we know that on yz-plane the coordinate of x is 0.

$\therefore \frac{\mathrm{m} \times 3+1 \times 2}{\mathrm{m}+1}=0$

$\Rightarrow 3 \mathrm{m}+2=0$

$\Rightarrow \mathrm{m}=-\frac{2}{3}$

Hence, yz-plane divide the line sgement joining the points (2, 4, 5) and (3, −5, 4) in 2 : 3 externally.

115. Write the coordinates of third vertex of a triangle having centroid at the origin and two vertices at (3, -5, 7) and (3, 0, 1).

Ans. : Let the coordinates of third vertex be (x_1, y_1, z_1)

Now,

$\frac{\mathrm{x}_1+3+3}{3}=0, \frac{\mathrm{y}_1+5+0}{3}=0$ and $\frac{\mathrm{z}_1+7+1}{3}=0$

$\Rightarrow \mathrm{x}_1=-6, \mathrm{y}_1=5$ and $\mathrm{z}_1=-8$

Hence, the coordinates of third vertex of a triangle is (−6, 5, −8).

*** Given section consists of questions of 2 marks each.** **[22]**

116. Show that the points (-2, 3, 5), (1, 2, 3) and (7, 0, -1) are collinear.

Ans. : Let A(-2, 3, 5), B (1, 2, 3) and C(7, 0, -1) be three given points.

Then $AB=\sqrt{(1+2)^2+(2-3)^2+(3-5)^2}=\sqrt{9+1+4}=\sqrt{14}$

$BC=\sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}=\sqrt{36+4+16}=\sqrt{56}=2\sqrt{14}$

$AC=\sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}=\sqrt{81+9+36}=\sqrt{126}=3\sqrt{14}$

Now AC = AB + BC

Therefore,A,B,C are collinear.

117. Find the equation of the set of points which are equidistance from the points (1, 2, 3) and (3, 2, -1).

Ans. :

Let a point P(x, y, z) be equidistant from the points A(1, 2, 3) and P(3, 2, -1).

Then, $PA=\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$

[$\because$ distance $=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2+\left(z_1-z_2\right)^2}$]

$=\sqrt{x^2-2x+1+y^2-4y+4+z^2-6z+9}$

and $PB=\sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$

[$\because$ distance $=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2+\left(z_1-z_2\right)^2}$]

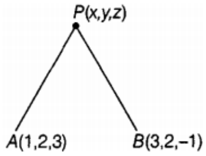
$=\sqrt{x^2-6x+9+y^2-4y+4+z^2+2z+1}$

$=\sqrt{x^2+y^2+z^2-6x-4y+2z+14}$

According to the question, PA = PB

$\therefore \sqrt{x^2+y^2+z^2-2x-4y-6z+14}$

$=\sqrt{x^2+y^2+z^2-6x-4y+2z+14}$



On squaring both sides, we get

$x^2 + y^2 + z^2 - 2x - 4y - 6z + 14 = x^2 + y^2 + z^2 - 6x - 4y + 2z + 14$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$ [dividing both sides by 4]

118. If the origin is the centriod of the triangle PQR with vertices P(2a, 2, 6), Q(-4, 3b, -10) and R(8, 14, 2c), then find the values of a, b and c.

Ans. : Here P(2a, 2, 6), $Q(-4, 3b, -10)$ and R(8, 14, 2c) are vertices of triangle PQR.

$\therefore$ Coordinates of centriod of ΔPQR is $\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)$

$= \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

But is it given that coordinates of centroid is (0, 0, 0)

$\frac{2a+4}{3} = 0 \Rightarrow$ 2a + 4 = 0 $\therefore$ a = -2

$\frac{3b+16}{3} = 0 \Rightarrow$ 3b + 16 = 0 $\Rightarrow b = \frac{-16}{3}$

$\frac{2c-4}{3} = 0 \Rightarrow$ 2c - 4 = 0 $\Rightarrow$ c = 2

119. Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P(3, -2, 5).

Ans. : Let Q (0, y, 0) be any point on y-axis.

$PQ = \sqrt{(0-3)^2 + (y+2)^2 + (0-5)^2}$

$= \sqrt{9 + y^2 + 4 + 4y + 25} = \sqrt{y^2 + 4y + 38}$

But $\sqrt{y^2 + 4y + 38} = 5\sqrt{2}$

Squaring both sides, we have

$y^2 + 4y + 38 = 50$

$\Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow (y - 2)(y + 6) = 0$

$\Rightarrow y = 2, -6$

Thus coordinates of point Q are (0, 2, 0) and (0, -6, 0)

120. A point R with x-coordinate 4 lies on the line segment joining the points P(2, -3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[Hint Suppose R divides PQ in the ratio k : 1. The coordinates of the point R are given by $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$].

Ans. : Let R (4, y, z) be any point which divides the join P(2, -3, 4) and Q (8, 0, 10) in the ratio k : 1 internally.

$\therefore$ Coordinates of R is $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$

But x coordinate of R is 4

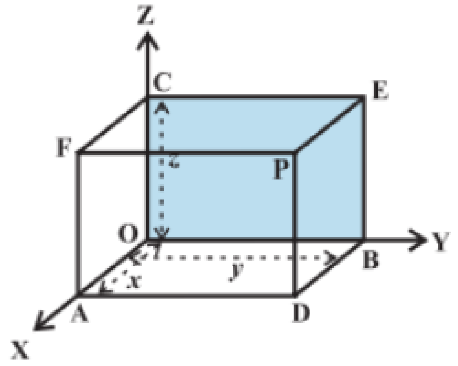
$\therefore \frac{8k+2}{k+1} = 4 \Rightarrow$ 8k + 2 = 4k + 5 $\Rightarrow k = \frac{1}{2}$

$\therefore y = \frac{-3}{\frac{1}{2}+1} = \frac{-3}{\frac{3}{2}} = -2$

$z = \frac{\frac{10\times1}{2}+4}{\frac{1}{2}+1} = \frac{9}{\frac{3}{2}} = 6$

Thus coordinates of R is (4, -2, 6).

121. In Fig, if P is (2,4,5), find the coordinates of F.



Ans. : For the point F, the distance measured along OY is zero.
Hence,, the coordinates of F are (2,0,5).

122. The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, – 6), respectively, find the coordinates of the point C.

Ans. : Let the coordinates of C be (x, y, z) and the coordinates of the centroid G be (1, 1, 1). Then

$\frac{x+3-1}{3} = 1, \Rightarrow$ x = 1

$\frac{y-5+7}{3} = 1, \Rightarrow$ y = 1

$\frac{z+7-6}{3} = 1, \Rightarrow$ z= 2

Hence, coordinates of C are (1, 1, 2).

123. The coordinates of a point are (3, -2, 5). Write down the coordinates of seven points such that the absolute values of their coordinates are the same as those of the coordinates of the given point.

Ans. : The seven coordinates are as follows:

(-3, -2, -5)
(-3, -2, 5)
(3, -2, -5)
(-3, 2, -5)
(3, 2, 5)
(3, 2, -5)
(-3, 2, 5)

124. Given that P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10) are collinear. Find the ratio in which Q divides PR.

Ans. : P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10)

$\mathrm{PQ}=\sqrt{4+4+4}=2\sqrt{3}$

$\mathrm{QR}=\sqrt{16+16+16}=4\sqrt{3}$

$\mathrm{PQ}:\mathrm{QR}=1:2$

125. Find the third vertex of triangle whose centroid is origin and two vertices are (2, 4, 6) and (0, -2, -5).

Ans. : Let the third or unknown vertex of $\triangle\mathrm{ABC}$ be A(x, y, z).
Other vertices of triangle are B(2,4, 6) and C(0, -2, -5).
The centroid is G(0, 0, 0).

$\therefore(0,0,0)=\left(\frac{2+0+x}{3},\frac{4-2+y}{3},\frac{6-5+z}{3}\right)$

On comparing coordinates, we get,

$\frac{2+x}{3}=0,\frac{2+y}{3}=0$ and $\frac{1+z}{3}=0$

$\Rightarrow x=-2,\ y=-2$ and $z=-1$

126. Show that if $x^2+y^2=1$, then the point $\left(x,y,\sqrt{1-x^2-y^2}\right)$ is at a distance 1 unit from the origin.

Ans. : Given that, $x^2+y^2=1$

$\therefore$ Distance of the point $\left(x,y,\sqrt{1-x^2-y^2}\right)$ from origin is given as

$$d=\sqrt{x^2+y^2+\left(\sqrt{1-x^2-y^2}\right)^2}=\sqrt{x^2+y^2+1-x^2-y^2}=1$$

*** Given section consists of questions of 3 marks each.** **[69]**

127. If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that PA^2 + PB^2 = k^2, where k is a constant.

Ans. : The equation of the set of points P such that PA^2 + PB^2 = k^2, where k is a constant
Given: The points A (3, 4, 5) and B (-1, 3, -7)
$\Rightarrow$ x_1 = 3, y_1 = 4, z_1 = 5; x_2 = -1, y_2 = 3, z_2 = -7;

PA^2 + PB^2 = k^2 ...(i)
Let the point be P (x, y, z).
Now, by Distance Formula, we know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by $\mathrm{PQ}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.
So, PA $=\sqrt{(3-x)^2+(4-y)^2+(5-z)^2}$
And PB $=\sqrt{(-1-x)^2+(3-y)^2+(-7-z)^2}$
Now, substituting these values in (i), we have
$[(3-x)^2+(4-y)^2+(5-z)^2]+[(-1-x)^2+(3-y)^2+(-7-z)^2]=k^2$

$\Rightarrow [(9 + x^2 - 6x) + (16 + y^2 - 8y) + (25 + z^2 - 10z)] + [(1 + x^2 + 2x) + (9 + y^2 - 6y) + (49 + z^2 + 14z)] = k^2$

$\Rightarrow 9 + x^2 - 6x + 16 + y^2 - 8y + 25 + z^2 - 10z + 1 + x^2 + 2x + 9 + y^2 - 6y + 49 + z^2 + 14z = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = k^2 - 109$

$\Rightarrow 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$

$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2-109}{2}$

128. Are the points A(3, 6, 9), B(10, 20, 30) and C(25, -41, 5), the vertices of a right-angled triangle?

Ans. : Given: A(3, 6, 9), B(10, 20, 30) and C(25, -41, 5),
According to the distance formula, we have

$AB^2 = (10 - 3)^2 + (20 - 6)^2 + (30 - 9)^2$
$= 49 + 196 + 441 = 686$

$BC^2 = (25 - 10)^2 + (-41 - 20)^2 + (5 - 30)^2$
$= 225 + 3721 + 625 = 4571$

and $CA^2 = (3 - 25)^2 + (6 + 41)^2 + (9 - 5)^2$
$= 484 + 2209 + 16 = 2709$

We observe that, $CA^2 + AB^2 \neq BC^2$
Hence, the $\triangle$ABC is not a right angled triangle.

129. Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, -8) is divided by the YZ-plane.

Ans. : Let YZ-plane divides the line segment joining the points A(4, 8, 10) and B(6, 10, -8) at P(x, y, z) in the ratio k: 1. Then, the coordinates of P are

$\left(\frac{4+6k}{k+1}, \frac{8+10k}{k+1}, \frac{10-8k}{k+1}\right)$

$\left[\begin{array}{l}\because \text{ coordinates of internal division,} \\ \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)\end{array}\right]$

Since P lies on the YZ-plane, its x-coordinate is zero,

i.e., $\frac{4+6k}{k+1} = 0 \quad \Rightarrow \quad k = -\frac{2}{3}$

Therefore, YZ-plane divides AB externally in the ratio 2:3.

130. The mid-points of the sides of a triangle ABC are given by (-2, 3, 5), (4, -1, 7) and (6, 5, 3). Find the coordinates of A, B and C.

Ans. : Given mid-points D(-2, 3, 5), E(4, -1, 7) and F(6, 5, 3)
Assume D is mid-point of AB, E is mid-point of BC
F is mid-point of CA

$A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$

From mid-point formula, we get following equations

$x_1 + x_2 = -4$; $x_2 + x_3 = 8$; $x_3 + x_1 = 12$

$y_1 + y_2 = 6$; $y_2 + y_3 = -2$; $y_3 + y_1 = 10$

$z_1 + z_2 = 10$; $z_2 + z_3 = 14$; $z_3 + z_1 = 6$

Solving above set of equations we get

A = (0, 9, 1)

B = (4, -3, 9)

C = (12, 1, 5)

131. The vertices of the triangle are A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2).The intenal bisector of angle A meets BC at D. Find the coordinates of D and the length AD.

Ans. : We know that angle bisector divides opposite side in ratio of other two sides

⇒ D divides BC in ratio of AB : AC

A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2)

$\mathrm{AB}\sqrt{16+25+9}=\sqrt{50}=5\sqrt{2}$

$\mathrm{AC}\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

AB : AC = 5 : 3 = m : n

$\mathrm{D}(\mathrm{x},\mathrm{y},\mathrm{z})=\left(\frac{\mathrm{mx}_2+\mathrm{nx}_1}{\mathrm{m}+\mathrm{n}},\frac{\mathrm{my}_2+\mathrm{ny}_1}{\mathrm{m}+\mathrm{n}},\frac{\mathrm{mz}_2+\mathrm{nz}_1}{\mathrm{m}+\mathrm{n}}\right)$

Substitute values for m : n = 5 : 3,

$(x_1, y_1, z_1) = (1, -1, 3)$

$(x_2, y_2, z_2) = (4, 3, 2)$

$\mathrm{D}=\left(\frac{23}{8},\frac{3}{2},\frac{19}{8}\right)$

$\therefore\ \mathrm{AD}=\sqrt{\left(5-\frac{23}{8}\right)^2+\left(4-\frac{12}{8}\right)^2+\left(6-\frac{19}{8}\right)^2}$

$=\sqrt{\frac{17^2+20^2+29^2}{8^2}}$

$=\sqrt{\frac{289+400+841}{8^2}}$

$=\frac{\sqrt{1530}}{8}$

132. Find the ratio in which the line joining (2, 4, 5) and (3, 5, 4) is divided by the yz-plane.

Ans. :

yz plane means x = 0

Given (2, 4, 5) and (3, 5, 4)

assume ratio to be m : n

lets equate x - term

$0=\frac{3\mathrm{m}+2\mathrm{n}}{\mathrm{m}+\mathrm{n}}$

3m = -2n

m : n = -2 : 3

which means yz plane divides the line in 2 : 3 ratio externally

133. Determine the point on z-axis which is equidistant from the points (1, 5, 7) and (5, 1, -4).

Ans. :

Let P(0, 0, z) be the equidistant from Q(1, 5, 7) and R(5, 1, -4).

So

$(PQ)^2 = (PR)^2 \Rightarrow (0 - 1)^2 + (0 - 5)^2 + (z - 7)^2 = (0 - 5) + (0 - 1)^2 + (z + 4)^2$

$\Rightarrow 1 + 25 + (z - 7)^2 = 25 + 1 + (z + 4)^2$

$\Rightarrow 26 + z^2 + 49 - 14z = 26 + z^2 + 8z + 16$

$\Rightarrow -14z - 8z = 16 - 49$

$\Rightarrow -22z = -33$

$\Rightarrow \mathrm{z} = \frac{-33}{-22}$

$\Rightarrow \mathrm{z} = \frac{3}{2}$

Required point $= \left(0,\ 0,\ \frac{3}{2}\right)$

134. A cube of side 5 has one vertex at the point(1, 0, -1) and the three edge from this vertex are, respectively, parallel to the negative x and y axes and positive z-axis. Find the coordinates of the other vertices of the cube.

Ans. : Let P ≡ (1, 0, -1)

The length of each side of the cube is 5.

The three edges from vertex of the cube are drawn from P towards the negative x and y axes and the positive z-axis.

Therefore, the coordinates of the vertex of the cube will be as follows:

x-coordinate = 1, 1-5 = -4, i.e. 1, -4

y-coordinate = 0, 0-5 = -5, i.e. 0, -5

z-coordinate = -1, -1 + 5 = 4, i.e. -1, 4

Hence, the remaining seven vertices of the cube are as follows:

(1, 0, 4)

(1, -5, -1)

(1, -5, 4)

(-4, 0, -1)

(-4, -5, 4)

(-4, -5, -1)

(4, 0, 4)

135. If the points A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6) are collinear, find the ratio in which C divides AB.

Ans. : A(3, 2, 4), B(9, 8, -10) and C(5, 4, -6)

$\mathrm{AC} = \sqrt{4 + 4 + 4} = 2\sqrt{3}$

$\mathrm{AB}=\sqrt{36+36+36}=6\sqrt{3}$

$\mathrm{BC}=\sqrt{16+16+16}=4\sqrt{3}$

$\mathrm{AC}:\mathrm{BC}=1:2$

136. Find the distances of the point P(-4, 3, 5) from the coordinate axes.

Ans. :

IMAGE

Let PQ be the perpendicular to the xy-plane and QA be perpendicular from Q to the y-axis.

PA will be perpendicular to the x-axis

Also, QA = |3| and PQ = |5|

Now, distance of P from the x-axis:

$\mathrm{PB}=\sqrt{\mathrm{BQ}^2+\mathrm{QP}^2}$

$=\sqrt{3^2+5^2}$

$=\sqrt{9+25}=\sqrt{34}$

Similarly,

From the right-angled $\triangle\mathrm{PAQ}$.

distance of P from the y-axis:

$\mathrm{PA}=\sqrt{\mathrm{AQ}^2+\mathrm{QP}^2}$

$=\sqrt{(-4)^2+(5)^2}$

$=\sqrt{16+25}=\sqrt{41}$

Similarly, the length of the perpendicular from P to the z-axis $=\sqrt{(-4)^2+(3)^2}$

$=\sqrt{16+9}$

$=\sqrt{25}=5$

137. A point C with z-coordinate 8 lies on the line segment joining the points A(2, -3, 4) and B(8, 0, 10). Find its coordinates.

Ans. :

z-coordinate 8

A(2, -3, 4) and B(8, 0, 10)

DR's of AB = (6, 3, 6)

DR's of BC = (x - 8, y - 0, 8 - 10)

Given A, B, C lie on same line

So values of DR's should be proportional

$\frac{\mathrm{x}-8}{6}=\frac{\mathrm{y}}{3}=\frac{8-10}{6}$

So x = 6, y = -1

point is (6, -1, 8)

138. Prove that the triangle formed by joining the three points whose coordinates are (1, 2, 3), (2, 3, 1) and (3, 1, 2) is an equilateral triangle.

Ans. :

Let the triangle formed be $\triangle \mathrm{ABC}$

$(\mathrm{AB})=\sqrt{(1-2)^2+(2-3)^2+(3-1)^2}$

$=\sqrt{(-1)^2+(-1)^2+(2)^2}$

$=\sqrt{6}$ units

$\mathrm{BC}=\sqrt{(2-3)^2+(3-1)^2+(1-2)^2}$

$=\sqrt{(-1)^2+(2)^2+(-1)^2}$

$=\sqrt{6}$ units

$\mathrm{AC}=\sqrt{(1-3)^2+(2-1)^2+(3-2)^2}$

$=\sqrt{(-2)^2+(1)^2+(1)^2}$

$=\sqrt{6}$ units

since, AB = BC = CA

So, $\triangle \mathrm{ABC}$ is an equilateral $\triangle$

139. Find the point on y-axis which is equidistant from the points (3, 1, 2) and (5, 5, 2).

Ans. :

Let P(0, y, 0) be a point on y-axis which is equidistant from Q(3, 1, 2) and R(5, 5, 2).

So

$(PR)^2 = (PQ)^2 \Rightarrow (0 - 5)^2 + (y - 5)^2 + (0 - 2)^2 = (0 - 3) + (y - 1)^2 + (0 + 2)^2$

$\Rightarrow 25 + y^2 + 25 - 10y + 4 = 9 + y^2 + 1 - 2y + 4$

$\Rightarrow -10y + 2y = 14 - 54$

$\Rightarrow -14z - 8z = 16 - 49$

$\Rightarrow -8y = -40$

$\Rightarrow y = 5$

so, the required point is (0, 5, 0)

140. Find the centroid of a triangle, mid-points of whose sides are (1, 2, -3), (3, 0, 1) and (-1, 1, -4).

Ans. :

(1, 2, -3), (3, 0, 1) and (-1, 1, 4)

Centroid of Traingle is given by

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

We know that

$x_1 + x_2 = 2$

$x_2 + x_3 = 6$

$x_1 + x_3 = -2$

Adding all gives $\Rightarrow 2(x_1 + x_2 + x_3) = 6$

so $x_1 + x_2 + x_3 = 3$

similarly, $y_1 + y_2 + y_3 = 3$; $z_1 + z_2 + z_3 = -6$

Centroid = (1, 1, -2)

141. Show that the three points A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10) are collinear and find the ratio in which C divides AB.

Ans. : If points are collinear then all points lie on same line
and DR's should be proportional
A(2, 3, 4), B(-1, 2, -3) and C(4, 1, -10)
DR's of AB = (3, 1, 7)
DR's of BC = (3, 1, 7)
So A, B, C are collinear
Length of AC $= \sqrt{36 + 4 + 196} = \sqrt{236}$
Length of AB $= \sqrt{9 + 1 + 49} = \sqrt{59}$
Ratio is AC : AB = 2 : 1
So C divides AB in ratio 2 : 1 externally

142. Find the points on z-axis which are at a distance $\sqrt{21}$ from the point (1, 2, 3).

Ans. :
Let P(0, 0, z) be at a distance of $\sqrt{21}$ from Q(1, 2, 3)
So
$\mathrm{PQ} = \sqrt{(0-1)^2 + (0-2)^2 + (\mathrm{z}-3)^2}$
$\sqrt{21} = \sqrt{(1)^2 + (2)^2 + (\mathrm{z}-3)^2}$
$21 - 5 = (\mathrm{z}-3)^2$
$16 = (\mathrm{z}-3)^2$
$\mathrm{z} - 3 = \pm 4$
$\mathrm{z} - 7 \text{ and } \mathrm{z} - -1$
So, the required points are (0, 0, 7) and (0, 0, -1)

143. The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, -5, 7) and (-1, 7, -6) respectively, find the coordinates of the point C.

Ans. :
Given Centroid (1, 1, 1)
A(3, -5, 7) and B(-1, 7, -6)
Equating terms, we get
$1 = \frac{3-1+x_3}{3}$
$1 = \frac{-5+7+y_3}{3}$
$1 = \frac{7-6+z_3}{3}$
$(x_3, y_3, z_3) = (1, 1, 2)$

144. A(1, 2, 3), B(0, 4, 1), C(-1, -1, -3) are the vertices of a triangle ABC. Find the point in which the bisector of the angle $\angle \mathrm{BAC}$ meets BC.

Ans. :

A(1, 2, 3), B(0, 4, 1), C(-1, -1, -3)

Angle bisctor at A divides BC in ratio of AB : AC

AB $= \sqrt{1+4+4} = 3$

AC $= \sqrt{14+9+36} = 7$

Assume D divides BC

m : n = 3 : 7

so D $= \left(\frac{-3}{10}, \frac{25}{10}, \frac{-2}{10}\right)$

145. Find the ratio in which the sphere $x^2 + y^2 + z^2 = 504$ divides the line joining the points (12, -4, 8) and (21, -9, 18).

Ans. :

(12, -4, 8) and (27, -9, 18)

Assume point P is dividing line in $\lambda : 1$ ratio, we get

$\mathrm{P} = \left(\frac{27\lambda+12}{\lambda+1}, \frac{-9\lambda+-4}{\lambda+1}, \frac{18\lambda+8}{\lambda+1}\right)$

P lies on Sphere, so substitute in Sphere equation

$x^2 + y^2 + z^2 = 504$

$9(9\lambda+4)^2 + (9\lambda+4)^2 + 4(9\lambda+4)^2 = 504(\lambda+1)^2$

$729\lambda^2 + 81\lambda^2 + 324\lambda^2 + 648\lambda + 72\lambda + 288\lambda + 144 + 16 + 64 = 504\lambda^2$
$+ 1008\lambda + 504$

$(1134 - 504)\lambda^2 + (1008 - 1008)\lambda + 224 - 504 = 0$

$630\lambda^2 = 280$

$\lambda^2 = \frac{4}{9}$

$\lambda = 2 : 3$

146. Planes are drawn parallel to the coordinate planes through the points (3, 0, -1) and (-2, 5, 4).
Find the lengths of the edges of the parallelopiped so formed.

Ans. :

IMAGE

Let P ≡ (3, 0, -1), Q ≡ (-2, 5, 4)

PE = Distance between the parallel planes ABCP and FQDE

= |4 + 1| = 5 (These planes are perpendicular to the z-axis)

PA = Distance between the parallel planes ABQF and PCDE

= |-2 - 3| = 5 (These planes are perpendicular to the x-axis)

Similarly, PC = |5 - 0| = 5

Thus, the length of the edges of the parallelepiped are 5, 5 and 5

147. Find the ratio in which the line Segment joining the points (2, -1, 3) and (-1, 2, 1) is divided by the plane x + y + z =5.

Ans. : (2, -1, 3) and (-1, 2, 1)

x + y + z = 5

Assume plane divides line in ratio $\lambda : 1$

so point P which is diving line in $\lambda : 1$ ratio is

$\mathrm{P} = \left(\frac{-\lambda+2}{\lambda+1}, \frac{2\lambda-1}{\lambda+1}, \frac{\lambda+3}{\lambda+1}\right)$

P lies on plane x + y + z = 5

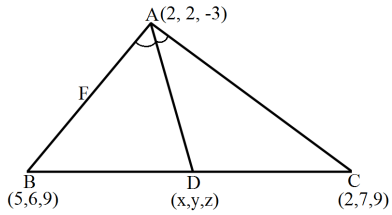
$-\lambda + 2 + 2\lambda - 1 + \lambda + 3 = 5\lambda + 5$

$3\lambda = -1 \Rightarrow \lambda = -1 : 3$

So plane diving line in 1 : 3 ratio externally

148. Let A(2, 2, -3), B(5, 6, 9) and C(2, 7, 9) be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D. Find the coordinates of D.

Ans. : Let the coordinates of D be (x, y, z).



$\mathrm{AB} = \sqrt{(5-2)^2 + (6-2)^2 + (9+3)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$

$\mathrm{AC} = \sqrt{(2-2)^2 + (7-2)^2 + (9+3)^2} = \sqrt{0+25+144} = \sqrt{169} = 13$

Thus, ABC is isosceles triangle with AB = AC.

So, angle bisector AD bisects BC or we can say that D is mid-point of BC.

$\therefore \mathrm{D} \equiv \left(\frac{5+2}{2}, \frac{6+7}{2}, \frac{9+9}{2}\right) \equiv \left(\frac{7}{2}, \frac{13}{2}, 9\right)$

149. Prove that the points (0, -1, -7), (2, 1, -9) and (6, 5, -13) are collinear. Find the ratio in which the first point divides the join of the other two.

Ans. : Let the given points are A(0, -1, -7), B(2, 1, -9) and C(6, 5, -13)

$\mathrm{AB} = \sqrt{(2-0)^2 + (1+1)^2 + (-9+7)^2}$

$= \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$

$\mathrm{BC} = \sqrt{(6-2)^2 + (5-1)^2 + (-13+9)^2}$

$= \sqrt{16+16+16} = \sqrt{48} = 4\sqrt{3}$

$\mathrm{AC} = \sqrt{(6-0)^2 + (5+1)^2 + (-13+7)^2}$

$= \sqrt{36+36+36} = \sqrt{108} = 6\sqrt{3}$

$2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$

i.e., $\mathrm{AB} + \mathrm{BC} = \mathrm{AC}$

$\therefore \mathrm{AB} : \mathrm{AC} = 2\sqrt{3} : 6\sqrt{3} = 1 : 3$

Hence, point A divides B and C in 1 : 3 externally.

*** Given section consists of questions of 5 marks each.** **[35]**

150. Determine the points zx-plane equidistant from the points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1).

Ans. :

Let R(x, 0, z) be the required point.

So

$(AR)^2 = (BR)^2 \Rightarrow (1 - x)^2 + (-1 - 0)^2 + (0 - z)^2 = (2 - x) + (1 - 0)^2 + (2 - z)^2$

$\Rightarrow 1 + x^2 - 2x + 1 + z^2 = 4 + x^2 - 4x + 1 + z^2 + 4z$

$\Rightarrow 2x + 4z = 7$ ... (i)

$(BR)^2 = (CR)^2 \Rightarrow (z - z)^2 + (1 - 0)^2 + (2 - z)^2 = (3 - x)^2 + (2 - 0)^2 (-1 - z)^2$

$\Rightarrow 4 + x^2 - 4x + 4 + z^2 - 4z = 9 + x^2 - 6x + 4 + 1 + z^2 + 2z$

$\Rightarrow 2x - 6z = 5$ ... (ii)

$(AR)^2 = (CR)^2 \Rightarrow (1 - x)^2 + (1 - 0)^2 + (0 - z)^2 = (3 - x)^2 + (2 - 0)^2 + (-1 - z)^2$

$\Rightarrow 1 + x^2 - 2x + 1 + z^2 = 9 + 6x + 4 + 1 + z^2$

$\Rightarrow 4x - 2z = 12$ ... (iii)

Solving equation (i) and (ii) we get

$\mathrm{z} = \frac{1}{5}, \mathrm{x} = \frac{31}{10}$

Put the value of x and z in equation (iii)

$4x - 2z = 12$

$4\left(\frac{31}{10}\right) - 2\left(\frac{1}{5}\right) = 12$

$\frac{124}{10} + \frac{2}{10} = 12$

$\frac{120}{10} = 12$

$12 = 12$

LHS = RHS.

so,

$\mathrm{x} = \frac{31}{10}, \mathrm{z} = \frac{1}{5}$

Required point $= \left(\frac{31}{10}, 0, \frac{1}{5}\right)$

151. Determine the points yz-plane equidistant from the points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1).

Ans. :

Let Q(0, y, z) be the required point.

So

$(AQ)^2 = (BQ)^2 \Rightarrow (0 - 1)^2 + (y + 1)^2 - (z - 0)^2 = (0 - 2) + (y + 1)^2 + (z - 2)^2$

$\Rightarrow 1 + y^2 + 1 + 2y + z^2 = 4 + y^2 + 1 - 2y + z^2 + 4 - 42$

$\Rightarrow 4y + 4z = 7$ ... (i)

$(BQ)^2 = (CQ)^2 \Rightarrow (0 - z)^2 + (y - 1)^2 + (z - 2)^2 = (0 - 3)^2 + (y - 2)^2 (2 + 1)^2$

$\Rightarrow 4 + y^2 + 1 - 2y + z^2 + 4 - 4z - 9 + y^2 + 4 - 4y + z^2 + 1 + 2z$

⇒ 2y - 6z = 5 ... (ii)

$(AQ)^2 = (CQ)^2 \Rightarrow (0 - 1)^2 + (y + 1)^2 + (z - 0)^2 = (0 - 3)^2 + (y - 2)^2 (z + 1)^2$

$\Rightarrow 1 + y^2 + 2y + 1 + z^2 = 9 + y^2 - 4y + 4 + z^2 + 1 + 2z$

⇒ 6y - 2z = 12 ... (iii)

Solving equation (i) and (ii) we get

$\mathrm{z} = \frac{-3}{16}$ and $\mathrm{y} = \frac{31}{16}$

Put the value of y and z in equation (iii)

6y - 2z = 12 = 12

$6\left(\frac{31}{16}\right) - 2\left(\frac{-3}{16}\right) = 12$

$\frac{186}{16} + \frac{6}{16} = 12$

$\frac{192}{16} = 12$

$12 = 12$

LHS = RHS.

so,

$\mathrm{y} = \frac{31}{16}, \mathrm{z} = \frac{13}{16}$

Required point $= \left(0, \frac{31}{16}, \frac{-3}{16}\right)$

152. If A(-2, 2, 3) and B(13, -3, 13) are two points. Find the locus of a point P which moves in such a way that 3PA = 2PB.

Ans. : Let P be $(x_1\ y_1\ z)$,

Here, A(-2, 2, 3), B(13, -3, 13) and 3PA = 2PB

$\Rightarrow 3\sqrt{(\mathrm{x}+2)^2(\mathrm{y}-2)^2+(\mathrm{z}-3)^2}$

$= 2\sqrt{(\mathrm{x}-13)^2+(\mathrm{y}+3)^2+(\mathrm{z}-13)^2}$

squaring both the sides,

$\Rightarrow 9[x^2 + 4x + 4 + y^2 + 4 - 4Y + z^2 + 9 - 6z]$

$= 4[x^2 + 169 - 26x + y^2 + 9 + 6y + z^2 + 169 - 26z]$

$\Rightarrow 9x^2 - 4x^2 + 36x + 104x + 36 - 676 + 9y^2 - 4y^2 + 36 - 36 - 36y - 24y + 9z^2 - 4z^2 + 81 - 676 - 54z + 6yz = 0$

$\Rightarrow 5x^2 + 5y^2 + 5z2 + 140x - 60y + 50z - 1235 = 0$

$\Rightarrow 5(x^2 + y^2 + z^2) + 140x - 60y + 50z - 1235 = 0$

153. Prove that the point A(1, 3, 0), B(-5, 5, 2), C(-9, -1, 2) and D(-3, -3, 0) taken in order are the vertices of a parallelogram. Also, show that ABCD is not a rectangle.

Ans. :

Here

$\mathrm{AB} = \sqrt{(1+5)^2+(3-5)^2+(0-2)^2}$

$= \sqrt{36+4+4}$

$= \sqrt{44}$

$= 2\sqrt{11}$ units

$\mathrm{BC} = \sqrt{(-5+9)^2 + (5+1)^2 + (2-2)^2}$

$= \sqrt{16+36}$

$= \sqrt{52}$

$= 2\sqrt{13}$ units

$\mathrm{CD} = \sqrt{(-9+3)^2 + (-1+3)^2 + (2-0)^2}$

$= \sqrt{36+4+4}$

$= 2\sqrt{11}$ units

$\mathrm{DA} = \sqrt{(-3-4)^2 + (-3-3)^2 + 0}$

$= \sqrt{16+36}$

$= \sqrt{52}$

$= 2\sqrt{13}$ units

$\mathrm{AC} = \sqrt{(1+9)^2 + (3+1)^2 + (0-2)^2}$

$= \sqrt{150+16+4}$

$= \sqrt{120}$

$= 4\sqrt{5}$ units

$\mathrm{BD} = \sqrt{(-3+5)^2 + (-3-5)^2 + (0-2)^2}$

$= \sqrt{4+64+4}$

$= \sqrt{72}$

$= 6\sqrt{2}$ units

Since,

AB = CD and BC = DA

⇒ ABCD is a parallelgram = BD

but, $\mathrm{AC} \neq \mathrm{BD}$

⇒ ABCD is not a rectangles.

154. Using distance formula prove that the following points are collinear:
P(0, 7, -7), Q(1, 4, -5) and R(-1, 10, -9)

Ans. :

P(0, 7, -7), Q(1, 4, -5) and R(-1, 10, -9)

$\mathrm{PQ} = \sqrt{(0-1)^2 + (7-4)^2 + (-7+5)^2}$

$= \sqrt{(1)^2 + (3)^2 + (-2)^2}$

$= \sqrt{1+9+4}$

$= \sqrt{14}$ units

$\mathrm{QR} = \sqrt{(1+1)^2 + (4-10)^2 + (-5+9)^2}$

$= \sqrt{(2)^2 + (-6)^2 + (4)^2}$

$= \sqrt{4+36+16}$

$= 2\sqrt{14}$ units

$PR = \sqrt{(0+1)^2 + (7-10)^2 + (-7+9)^2}$

$= \sqrt{1^2 + (-3)^2 + (2)^2}$

$= \sqrt{1+9+4}$

$= \sqrt{14}$ units

Since, PQ + PR = QR

so, P, Q, R are collinear.

155. Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of an isosceles right-angled triangle.

Ans. :

Let A = (0, 7, 10), B = (-1, 6, 6) and C = (-4, 9, 6)

$AB = \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2}$

$= \sqrt{(1)^2 + (1)^2 + (4)^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$ units

$BC = \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2}$

$= \sqrt{(3)^2 + (3)^2 + (0)}$

$= \sqrt{18}$

$= 3\sqrt{2}$ units

$AC = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2}$

$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$

$= \sqrt{36}$

$= 6$ units

$(AB)^2 + (BC)^2$

$= (3\sqrt{2})^2 + (3\sqrt{2})^2$

$= 18 + 18$

$= 36$

$= (AC)^2$

Also (AB) = (BC)

Hence (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of an isosceles right-angled triangle.

156. Using distance formula prove that the following points are collinear:
A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)

Ans. :

A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)

$AB = \sqrt{(3+1)^2 + (-5-0)^2 + (1-8)^2}$

$= \sqrt{(4)^2 + (-5)^2 + (-7)^2}$

$= \sqrt{16+25+49}$

$= \sqrt{90}$

$= 3\sqrt{10}$ units

$\mathrm{BC} = \sqrt{(-1-7)^2+(0+10)^2+(8+6)^2}$

$= \sqrt{(-8)^2+(10)^2+(14)^2}$

$= \sqrt{64+100+196}$

$= \sqrt{360}$

$= 6\sqrt{10}$ units

$\mathrm{CA} = \sqrt{(3-7)^2+(-5+10)^2+(1+6)^2}$

$= \sqrt{(-4)^2+(5)^2+(7)^2}$

$= \sqrt{16+25+49}$

$= \sqrt{90}$

$= 3\sqrt{10}$ units

Since, AB + AC = BC

so, A, B and C are collinear.

----- हर कोशिश में शायद सफलता नहीं मिल पाती,लेकिन हर सफलता का कारण कोशिश ही होती है| -----